**T-79.3001 Logic in Computer Science: Foundations** **Spring 2009**
**Exercise 10 ([Nerode and Shore, 1997], Predicate Logic, Chapters 6 – 7 )**
**April 16 – 20, 2009**

## Solutions to demonstration problems

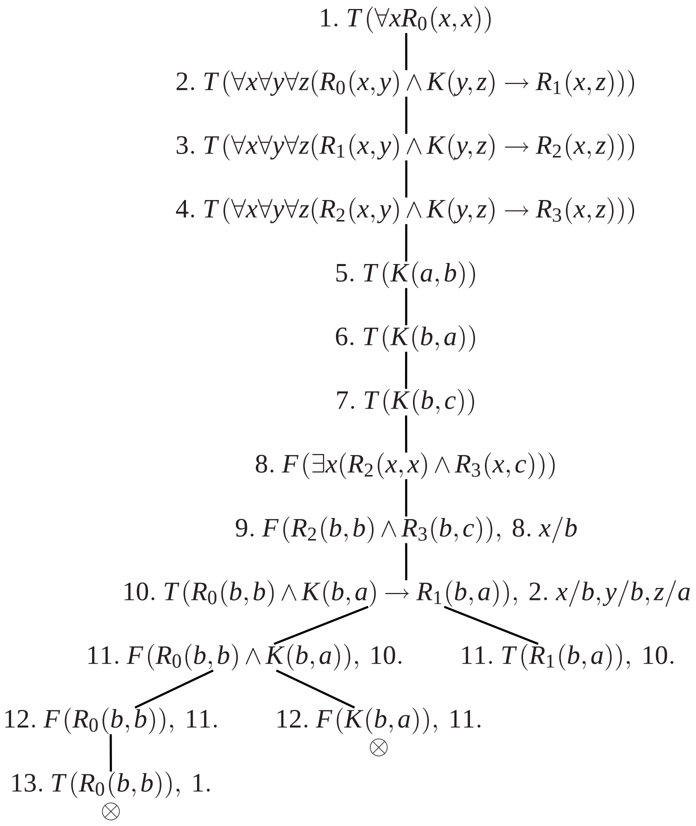
### Solution to Problem 7

a) Define predicates $R_n(x,y)$ as follows:

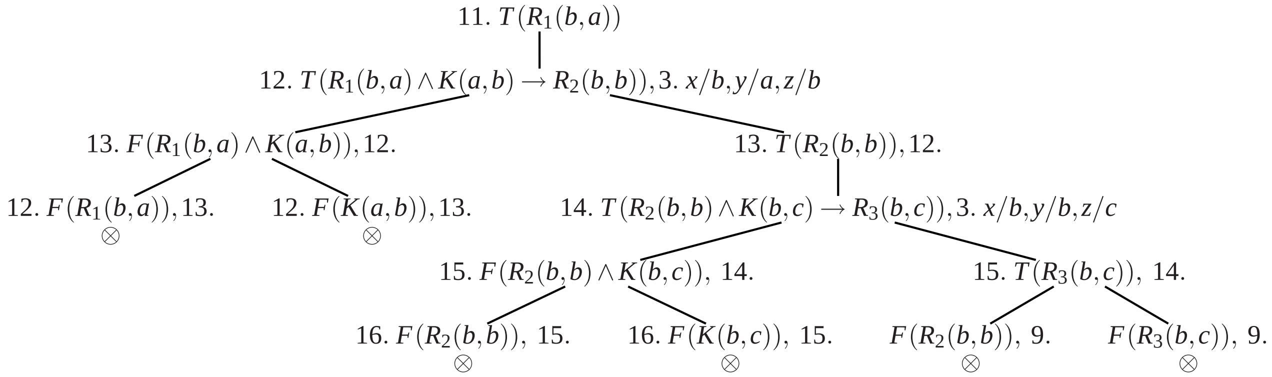
$$\begin{gathered}\forall x R_0(x,x)\\ \forall x\forall y\forall z(R_0(x,y)\wedge K(y,z)\rightarrow R_1(x,z))\\ \forall x\forall y\forall z(R_1(x,y)\wedge K(y,z)\rightarrow R_2(x,z))\\ \vdots\\ \forall x\forall y\forall z(R_{k-1}(x,y)\wedge K(y,z)\rightarrow R_k(x,z))\end{gathered}$$

The graph can be represented as $K(a,b)$, $K(b,a)$ and $K(b,c)$.

b)



The finished tableau is contradictory and the claim holds.



The subtree from node 11 continues in the next page.

**Solution to Problem 8**

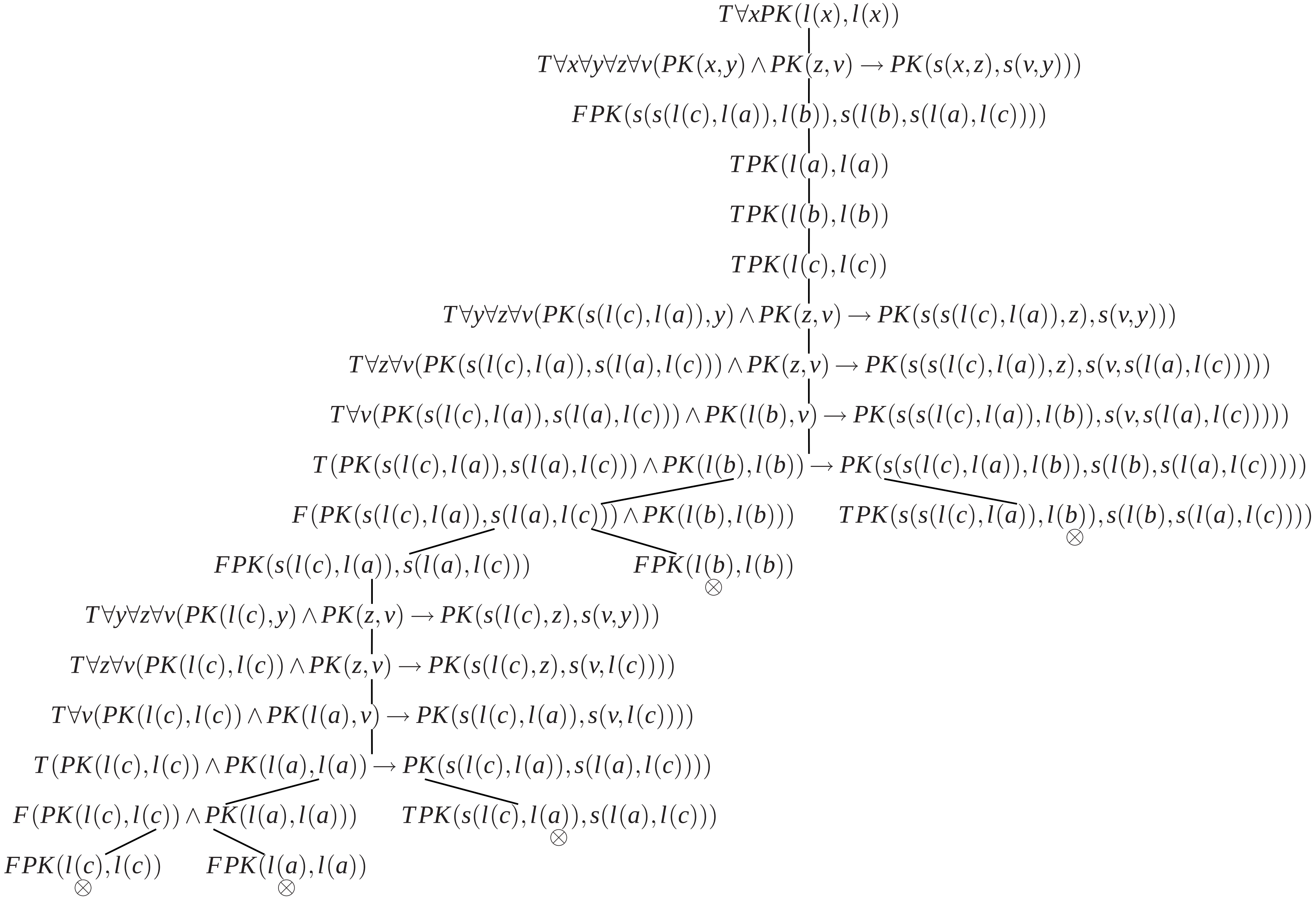
Define predicate $PK$ as follows:

$$\begin{aligned}&\forall x PK(l(x), l(x))\\&\forall x\forall y\forall v\forall w(PK(x,v)\wedge PK(y,w)\rightarrow PK(s(x,y),s(w,v)))\end{aligned}$$

We show that

$$PK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$$

is a logical consequence of the definition of $PK$ with the semantic tableau given in the next page.


$T\forall x PK(l(x),l(x))$
$T\forall x\forall y\forall z\forall v(PK(x,y)\wedge PK(z,v)\rightarrow PK(s(x,z),s(v,y)))$
$FPK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$
$TPK(l(a),l(a))$
$TPK(l(b),l(b))$
$TPK(l(c),l(c))$
$T\forall y\forall z\forall v(PK(s(l(c),l(a)),y)\wedge PK(z,v)\rightarrow PK(s(s(l(c),l(a)),z),s(v,y)))$
$T\forall z\forall v(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(z,v)\rightarrow PK(s(s(l(c),l(a)),z),s(v,s(l(a),l(c)))))$
$T\forall v(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),v)\rightarrow PK(s(s(l(c),l(a)),l(b)),s(v,s(l(a),l(c)))))$
$T(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),l(b))\rightarrow PK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c)))))$
$F(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),l(b)))$
$TPK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$
$\otimes$
$FPK(s(l(c),l(a)),s(l(a),l(c)))$
$FPK(l(b),l(b))$
$\otimes$
$T\forall y\forall z\forall v(PK(l(c),y)\wedge PK(z,v)\rightarrow PK(s(l(c),z),s(v,y)))$
$T\forall z\forall v(PK(l(c),l(c))\wedge PK(z,v)\rightarrow PK(s(l(c),z),s(v,l(c))))$
$T\forall v(PK(l(c),l(c))\wedge PK(l(a),v)\rightarrow PK(s(l(c),l(a)),s(v,l(c))))$
$T(PK(l(c),l(c))\wedge PK(l(a),l(a))\rightarrow PK(s(l(c),l(a)),s(l(a),l(c))))$
$F(PK(l(c),l(c))\wedge PK(l(a),l(a)))$
$TPK(s(l(c),l(a)),s(l(a),l(c)))$
$\otimes$
$FPK(l(c),l(c))$
$\otimes$
$FPK(l(a),l(a))$
$\otimes$

**Solution to Problem 9**

Let predicate $K(x)$ denote that $x$ is Father Christmas and predicate $J(x)$ denote that $x$ is Santa Claus. Thus we get the following sentences:

1. $\exists x K(x) \wedge \forall x \forall y (K(x) \wedge K(y) \rightarrow x = y)$,
2. $\forall x (J(x) \rightarrow K(x))$, ja
3. $\forall x (K(x) \rightarrow J(x))$.

Sentence 4 is of the form: $\exists x J(x) \wedge \forall x \forall y (J(x) \wedge J(y) \rightarrow x = y)$. The semantic tableau: