**T-79.3001 Logic in Computer Science: Foundations Spring 2009**
**Exercise 9 ([Nerode and Shore, 1997], Predicate Logic, Chapters 4 and 9)**
**April 2 – 6, 2009**

## Tutorial problems

**1.** Show using semantic tableaux:

a) $\{\forall x(P(x) \to R(x)), \forall x(\neg Q(x) \to \neg R(x))\} \models \forall x(P(x) \to Q(x))$.

b) $\models \exists x \exists y(P(x,y) \to \forall x \forall y P(x,y))$.

**2.** Examine the given statement and, if it does not hold, give a structure as a justification:

a) $\{\exists x(A(x) \leftrightarrow \neg B(x))\} \models \forall x \neg(A(x) \wedge B(x))$.

b) $\models \forall x \exists y(P(x) \wedge Q(y)) \to \exists y \forall x(P(x) \to Q(y))$.

**3.** Let us define the predicate $B(x,y)$ ($x$ is boss of $y$), $S(x,y)$ ($x$'s salary is greater than or equal to $y$'s), and $EQ(x,y)$ ($x$ and $y$ have the same salary) as follows:

$$\forall x \forall y(B(x,y) \to S(x,y))$$
$$\forall x \forall y \forall z(S(x,y) \wedge B(y,z) \to S(x,z))$$
$$\forall x \forall y(S(x,y) \wedge S(y,x) \to EQ(x,y))$$

Write a database such that Anu is Bob's and David's boss, Bob is David's boss, David is Cecilia's boss and Cecilia is Bob's boss. Show that Bob and Cecilia have equal salaries using semantic tableaux.

## Demonstration problems

**4.** Use semantic tableaux to see whether the following claims holds.

a) $\{\forall x \exists y(P(x) \to Q(y)), \forall x P(x)\} \models \forall x Q(x)$

b) $\{\forall x \forall y(\exists z(R(x,z) \wedge R(z,y)) \to R(x,y)), R(a,b), R(b,a)\} \models R(a,a)$

c) $\models \forall x \exists y R(x,y) \to (\forall y(\neg S(y) \to \neg \exists x R(x,y)) \to \exists x S(x))$

**5.** We know that

(i) All guilty persons are liars.

(ii) At least one of the accused is also a witness.

(iii) No witness lies.

Use semantic tableaux to prove that all accused are not guilty.

**6.** We know that:

1) If a brick is on another brick, then it is not on the table.
2) Every brick is either on the table or on another brick.
3) No brick is on a brick which is also on some other brick.

Use semantic tableaux to prove that if a brick is on another brick, the other brick is on the table.