T-79.3001 Logic in Computer Science: Foundations **Spring 2009**
Exercise 7 ([Nerode and Shore, 1997], Predicate Logic, Chapters 1–4)
March 19 – March 23, 2009

Tutorial problems

1. Formalize the following statements in predicate logic:

a) Vehicle is any device with wheels, caterpillar treads or skids which moves on the ground and does not move on rails; and a scrap vehicle is any vehicle with low or nonexistent value.

b) Someone unsatisfied with the transfer compensation is allowed to appeal against it to the County Administrative Court of that county where the transfer took place.

c) When applying the legal provisions of this law, the items in the vehicle are considered as part of the vehicle. Items in a scrap vehicle are, however, considered as lost property unless they are meant to be used permanently in the vehicle.

Draw the syntax trees for the sentences.

2. The theory of recursive data structures (RDS) describes a set of data structures that are ubiquitous in programming. For example in C, **struct** defines a simple RDS where a single variable has multiple fields. Truly recursive data structures include lists, stacks and binary trees.

The following definitions are related to the theory of lists. We define them using three function symbols $(cons, car, cdr)$, and a predicate symbol (Atom). The functions are defined as follows:

- $cons$ is a list constructor: $cons(a,b)$ represents a list obtained by concatenating a to b,
- car is the left projection: $car(cons(a,b)) = a$, and
- cdr is the right projection: $cdr(cons(a,b)) = b$.

The unary predicate $\text{Atom}(x)$ is true iff x is a single-element list.

Explain in English the following propositions related to the theory of RDS:

a)

$$\forall x_1 \forall x_2 \forall y_1 \forall y_2 (x_1 = x_2 \land y_1 = y_2 \rightarrow cons(x_1,y_1) = cons(x_2,y_2))$$
$$\forall x \forall y (x = y \rightarrow car(x) = car(y))$$
$$\forall x \forall y (x = y \rightarrow cdr(x) = cdr(y))$$

b) $\forall x \forall y (car(cons(x,y)) = x)$

c) $\forall x (\neg \text{Atom}(x) \rightarrow cons(car(x), cdr(x)) = x)$

3. A graph is a set S of nodes and a set E of edges between the nodes ($E \subseteq S \times S$). A set of nodes $P \subseteq S$ is a *clique* for the graph, iff it holds for all $s, s' \in P$ where $s \neq s'$ that $\langle s, s' \rangle \in E$. The *clique problem* is to find a clique for a graph.

a) Formalize the clique problem using predicate logic.

b) Give a model for your formalization.

c) Give a structure that doesn't satisfy your formalization.

Demonstration problems

4. Formalize the following sentences using predicate logic:

a) There is a faulty gate.

b) This algorithm is the fastest.

c) Each participant of this course has a workstation to use

d) Only one process can write in each file at a time

Draw the syntax trees for sentences a) and b).

5. Remove unnecessary parenthesis so that the meaning of statement does not change.

a) $(\forall y((\exists x(P(x) \wedge Q(x))) \rightarrow L(y)))$

b) $((\exists x(\exists y(P(x,y) \vee Q(y,x)))) \leftrightarrow (\forall x(\neg K(f(x)))))$

c) $(\forall x(\forall y(A \wedge B)))$

6. What ground (variable-free) terms can you compose from a constant c, a unary function symbol f and a binary function symbol g?

7. Represent arbitrary trees with function symbols using at most three constant or function symbols.

8. Show that if $\forall x \phi(x)$ is a sentence and t is a ground term, then $\phi(t)$ is a sentence.

9. Consider a domain $\mathbb{N}^2 = \{\langle x, y \rangle | x \in \mathbb{N}, y \in \mathbb{N}\}$. Choose interpretations for a constant c and a unary function symbol $f \in \mathcal{F}_1$ such that each element in the domain has an interpretation.

10. A graph is a set S of nodes and a set K of edges between the nodes ($K \subseteq S \times S$). The nodes s and s' of the graph are adjacent, if they are connected with an edge ($\langle s, s' \rangle \in K$). Let C be a set of colors. The problem of *node coloring* is to find a color in C for each node of the graph so that each node has a unique color and two adjacent nodes have different colors.

a) Formalize the node coloring problem using predicate logic.

b) Give a model for your formalization.

c) Give a structure that doesn't satisfy your formalization.