

Tutorial problems

1. Use resolution to show that

$$\{a \vee b \vee c, a \rightarrow (b \leftrightarrow c), c \rightarrow a \vee b, b \rightarrow a \vee c, a \rightarrow b \vee c\} \models b \wedge c.$$

2. Show the above using semantic tableaux.
3. Show that an arbitrary set of clauses S can be reduced to a clause set $r(S)$ such that $r(S)$ contains clauses with at most three literals and $S \in \text{SAT}$ if and only if $r(S) \in \text{SAT}$.

Demonstration problems

4. A few weeks ago a traffic light system was modeled. Transform the propositions specifying the behaviour of the system into clauses and prove with resolution that both red lights are not on at the same time.
5. One successful application of expert systems has been analyzing the problem of which chemical syntheses are possible. Consider the following chemical reactions:
 - (1) $\text{MgO} + \text{H}_2 \rightarrow \text{Mg} + \text{H}_2\text{O}$
 - (2) $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$
 - (3) $\text{CO}_2 + \text{H}_2\text{O} \rightarrow \text{H}_2\text{CO}_3$
 - a) Represent these rules and the assumptions that we have some MgO , H_2 , O_2 and C by propositional logic formulas.
 - b) Give a resolution proof that we can get some H_2CO_3 .
6. Construct a deterministic Turing machine that counts the successor of a given binary number.
7. Show the problem of deciding whether there exists a 3-coloring for a graph is in the class **NP** by reducing it into the propositional satisfiability problem.