T-79.3001 Logic in Computer Science: Foundations **Spring 2009**
Exercise 5 ([Nerode and Shore, 1997], Chapter I, Sections 4 and 7)
February 19 – February 23, 2009

Tutorial problems

1. Find disjunctive and conjunctive normal forms for the following propositions using the transformation rules.

a) $(A \vee B) \wedge (\neg(C \vee D) \vee \neg E)$.

b) $(A \rightarrow B \wedge \neg C) \vee (B \wedge \neg C)$.

2. Find disjunctive and conjunctive normal forms for the following propositions using semantic tableaux.

a) $\neg(C \rightarrow A) \rightarrow B$.

b) $(B \rightarrow \neg A) \rightarrow \neg B \vee \neg A$.

3. Find the clausal form for $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$. Give a truth assignment $\mathcal{A}$ such that it is a model for the resulting set of clauses.

Demonstration problems

4. Find disjunctive and conjunctive normal forms for the following propositions using (1) transformation rules and (2) semantic tableaux.

a) $A \rightarrow (B \rightarrow C)$.

b) $\neg A \leftrightarrow ((A \vee \neg B) \rightarrow B)$.

c) $\neg((A \leftrightarrow \neg B) \rightarrow C)$.

d) $A \wedge B \leftrightarrow (A \rightarrow B) \vee (B \rightarrow C)$.

5. Use semantic tableaux to prove that the rules used to derive CNF/DNF of a proposition preserve logical equivalence.

6. Find CNFs for the following propositions both using transformation rules and semantic tableaux.

a) $(P \wedge \neg P) \vee (Q \wedge \neg Q)$.

b) $(P_1 \wedge \neg P_1) \vee \cdots \vee (P_n \wedge \neg P_n)$.

Use semantic tableaux to prove that CNF obtained for a) is unsatisfiable.

7. Find a clausal form for

$$(A \to ((A \to A) \to A)) \to ((A \to (A \to A)) \to (A \to A)).$$

8. Consider the following set of clauses:

$$\begin{aligned} S \quad = \quad & \{\{A_0, A_1\}, \{\neg A_0, \neg A_1\}, \{A_1, A_2\}, \{\neg A_1, \neg A_2\}, \ldots, \\ & \{A_{n-1}, A_n\}, \{\neg A_{n-1}, \neg A_n\}, \{A_n, A_0\}, \{\neg A_n, \neg A_0\}\}. \end{aligned}$$

Give truth assignment $\mathcal{A}$ such that $\mathcal{A} \models S$.

9. A Horn clause is a clause that has exactly one positive literal. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be models for a set of Horn-clauses S. Show that also $\mathcal{A} = \mathcal{A}_1 \cap \mathcal{A}_2$ is a model of S.