T-79.3001 Logic in Computer Science: Foundations **Spring 2009**
Exercise 4 ([Nerode and Shore, 1997], Chapter I, Sections 4 and 7)
February 19 – February 23, 2009

Tutorial problems

1. Use semantic tableaux to check whether the following claims hold. If not, give a counter-example.

a) $\{A \rightarrow B, B \rightarrow C, C \rightarrow A\} \models (A \leftrightarrow C)$.

b) $\not\models ((A \rightarrow B) \rightarrow C) \leftrightarrow (A \rightarrow (B \rightarrow C))$.

c) $A \wedge B \wedge (B \rightarrow C) \wedge (\neg A \vee \neg B \vee \neg C)$ is unsatisfiable.

2. Use semantic tableaux to prove the axioms of the Hilbert system. Use propositional variables α, β and γ instead of atomic propositions.

3. Use the Suppes system to prove the axioms of the Hilbert system. Use propositional variablse instead of atomic propositions.

Demonstration problems

4. Use the Hilbert system to prove that

$$\{B \rightarrow A, \neg A\} \vdash \neg B.$$

5. Peirce arrow is defined as follows:

$$A \downarrow B \Leftrightarrow_{def} \neg A \wedge \neg B.$$

Define semantic tableaux rules for it.

6. Use semantic tableux to show that the following propositions are valid.

a) $A \rightarrow (B \rightarrow B)$.

b) $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$.

c) $(A \rightarrow B) \wedge (A \rightarrow C) \rightarrow (A \rightarrow B \wedge C)$.

d) $(A \rightarrow C) \wedge (B \rightarrow C) \wedge (A \vee B) \rightarrow C$.

7. Use semantic tableaux to check whether the following claims hold. If not, give a counter-example.

a) $\{B \rightarrow A, C \rightarrow B, (C \rightarrow A) \rightarrow D\} \models D$.

b) $\{A \rightarrow C, A \vee B, \neg D \rightarrow \neg B\} \models C \rightarrow D$.

c) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$.

d) $\models (\neg B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \vee C))$.

8. Recall the specification for two traffic light posts positioned in the intersection of two one-way streets discussed earlier in tutorials. Use semantic tableaux to prove that “the red lights cannot be on simultaneously” is a logical consequence of the set of propositions describing the behavior of the system.

9. Use Hilbert’s proof system to prove the following.

a) $\vdash P \rightarrow P$.

b) $\{P \rightarrow Q, Q \rightarrow R\} \vdash P \rightarrow R$.

c) $\{P, Q \rightarrow (P \rightarrow R)\} \vdash Q \rightarrow R$.