T-79.3001 Logic in Computer Science: Foundations **Spring 2009**
Exercise 2 ([Nerode and Shore, 1997], Chapter I, Sections 2 and 3)
February 5 – February 9, 2009

Tutorial problems

1. Give definitions for other propositional connectives using conjunction $(\wedge)$ and negation $(\neg)$.

2. a) Let $\mathcal{A} = \{A, C\}$ be a truth assignment. Find the truth value of

$$C \wedge (\neg A \leftrightarrow B) \rightarrow ((\neg A \vee \neg B) \wedge (B \vee A) \rightarrow C)$$

using (i) truth tables and (ii) the truth definition of propositional logic. What can be said about the validity, satisfiability, and unsatisfiability of the proposition?

b) Apply truth tables to see whether $\{A \rightarrow B, \neg(\neg C \wedge B)\} \models A \rightarrow C$ holds.

3. Show that if the set Σ of sentences has a unique model $\mathcal{A}$, then for every proposition ϕ, either $\Sigma \models \phi$ or $\Sigma \models \neg\phi$ (but not both), and

$$\mathrm{Cn}(\Sigma) = \{\phi \in \mathcal{L} \mid \mathcal{A} \models \phi\}.$$

Demonstration problems

4. Let $\mathcal{A} = \emptyset$ be a truth assignment. Find the truth value of the sentence

$$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$$

using a truth table and the truth definition of propositional logic.

5. Give definitions for propositional connectives using

a) the proposition that is always false $(\bot)$ and implication $(\rightarrow)$, and

b) the Sheffer stroke.

6. List all possible binary connectives (16 in total) and give their definitions using the basic connectives of propositional logic.

7. Define the Sheffer's stroke using the Peirce's arrow.

8. An engineer designed a specification for two traffic light posts positioned in the intersection of two one-way streets:

(i) Both the light posts have a green, a yellow and a red light. Exactly one of the lights in each light post is lit at all times.

(ii) Both green lights are not lit at the same time.

(iii) If one lamp post has the red light on, then the other has either the green or the yellow light on.

a) Formalize the above requirements as a set of propositional statements.

b) Construct a truth table for the set of statements.

c) Give (i) a model for the set of statements, and (ii) a truth assignment such that the set of statements is not satisfied.

d) Are the requirements complete enough for a real life situation?

9. Apply truth tables to see whether the following claims hold.

a) $(A \to B) \to ((B \to C) \to (A \to C))$ is valid.

b) $\neg((A \to B) \to ((\neg A \to B) \to B))$ is unsatisfiable.

c) $A \leftrightarrow B$ and $\neg(A \leftrightarrow \neg B)$ are logically equivalent.

d) $\{(A \wedge B) \vee (C \wedge A), (A \wedge B) \vee \neg B\} \models A \vee (C \wedge \neg B)$.

10. A sentence ϕ is called *positive*, if it has been formed using only atomic propositions and the connectives $\wedge$ and $\vee$. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be truth assignments so that $\mathcal{A}_1 \subseteq \mathcal{A}_2$.

a) Show using induction on the structure of a positive sentence ϕ that if $\mathcal{A}_1 \models \phi$, then $\mathcal{A}_2 \models \phi$.

b) Explain why all propositional sentences do not have this property. Give a concrete counter-example.

11. Let $\mathcal{A}_1 \subseteq \mathcal{P}$ and $\mathcal{A}_2 \subseteq \mathcal{P}$ be truth assignments and $\phi \in \mathcal{L}$ a proposition. Show that if $\mathcal{A}_1 \cap At(\phi) = \mathcal{A}_2 \cap At(\phi)$, then $\mathcal{A}_1 \models \phi \iff \mathcal{A}_2 \models \phi$.