T-79.3001 Logic in Computer Science: Foundations **Spring 2009**
Exercise 12 (Recap)
May 4 – May 4, 2009

Tutorial problems

1. a) Define the following concepts: *theorem*, *contradictory path*, and *structure*.

b) What is meant by the notation $\phi \equiv \psi$?
Prove in detail that if $\phi \equiv \psi$, then $\phi \wedge \chi \equiv \psi \wedge \chi$ for any sentence χ.

2. Prove the following claims using semantic tableaux:

a) $\not\models ((A \to B) \to C) \leftrightarrow (A \to (B \to C))$

b) $\{\forall x(P(x) \to R(x)), \forall x(\neg Q(x) \to \neg R(x))\} \models \forall x(P(x) \to Q(x))$

Tableau proofs must contain all intermediary steps!

3. Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x(A(x) \vee B(x)) \to \exists x A(x) \vee \exists x B(x)).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Demonstration Problems

4. Let us consider a stack of books which is described using a binary predicate $T(x,y)$ = "book x is immediately on top of book y in the stack". Suppose that b, c, and d are three constants referring to specific books authored by Böll, Carr, and Dostojevski, respectively.

a) Define a ternary predicate $B(x,y,z) =$ "book y appears between books x and z, out of which x appears higher in the stack than y and z" using predicate logic so that your definition covers all books in an individual stack.

b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \cup \{T(b,c), T(c,d)\} \not\models B(d,c,b).$$

5. Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then }\{C_1\}\texttt{ else }\{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

$$\texttt{v=x ; z=y ; while(!(z==0))}\ \{\texttt{z=z-1 ; v=v-1}\}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==x-y}].$$