T-79.3001 Logic in computer science: foundations **Spring 2009**
Exercise 1 ([Nerode and Shore, 1997], Chapter I, Sections 1 and 2)
January 29–February 2, 2009

Tutorial problems

1. a) Show by structural induction that the depth of the parse tree of a propositional formula with n connectives is at most n.

b) Give an example of a propositional formula with n connectives having a parse tree of depth n for arbitrary $n \geq 0$.

2. Formalize the following statements in propositional logic:

a) Greasy fish is served with a white wine with high acidity. Otherwise the dish is served with still or sparkling mineral water.

b) If the customer chooses a cabriolet car, the roof rack cannot be installed.

c) A car has exactly 4, 5, or 6 gears.

d) The course has three weekly lectures each organized different day.

c) Out of two three-digit binary numbers $x_1x_2x_3$ is greater than $y_1y_2y_3$.

3. Remove unnecessary parenthesis from the following propositional statements. What are the forms of the statements? Give parse trees for the propositions.

a) $(((C \rightarrow (\neg B)) \vee A) \wedge ((\neg A) \leftrightarrow D))$

b) $((\neg(A \rightarrow (B \vee (\neg D)))) \rightarrow ((\neg B) \vee (C \vee (\neg A))))$

c) $(A \leftrightarrow (D \vee ((B \rightarrow (\neg D)) \wedge C)))$

Demonstration problems

4. Let $\mathcal{P} = \{A, B, C\}$ be the set of atomic propositions. Which of the following are propositional statements? Why?

a) A

b) $\neg(A \wedge B))$

c) $(A \wedge (B \rightarrow (A \wedge C)))$

d) It is raining today.

5. Formalize the following statements in propositional logic:

a) I can't finish my work unless you help me.

b) I either walk, ride a bicycle, or sometimes drive a car to work.

c) Merja and Arto are coming to visit us.

d) You won't get dessert because you have been naughty

e) Even though the manual was long I finished reading it too early.

f) If somebody asks me — or even if no one does — he shouldn't buy a car or he must live far from his workplace and gasoline should become cheaper.

6. Remove unnecessary parenthesis so that the meaning of the proposition does not change.

a) $(A \to ((B \land C) \lor D))$

b) $(((A \to B) \land (B \to C)) \to (A \to C))$

c) $((A \land (B \lor C)) \lor (A \land (C \lor D)))$

d) $((\neg(A \land B)) \leftrightarrow ((B \to C) \land A))$

e) $(((\neg A) \land (\neg B)) \to \neg(A \lor B))$

7. What are the forms of the propositional statements in the previous exercise? Give parse trees for the propositions.

8. List the substatements of the following propositional statement.

$$(\neg A \to (\neg B \to C)) \to (\neg(\neg A \to B) \to C)$$

9. Prove by induction that a set of n elements has 2^n subsets.

10. Prove that all propositional statements have an even number of parenthesis.

11. The set of atomic propositions of an arbitrary proposition Φ is denoted by $\mathrm{At}(\Phi)$. Write a recursive definition for $\mathrm{At}(\Phi)$.