T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 10 ([NS, 1997], Predicate Logic, Chapters 6 – 7)
April 16–18, 2008

Solutions to demonstration problems

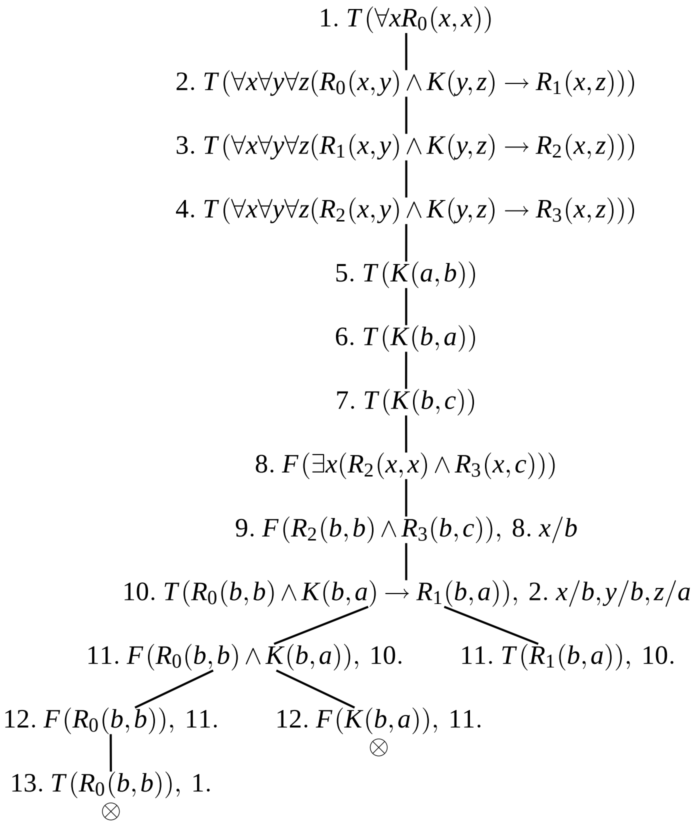
Solution to Problem 4

a) Define predicates $R_n(x,y)$ as follows:

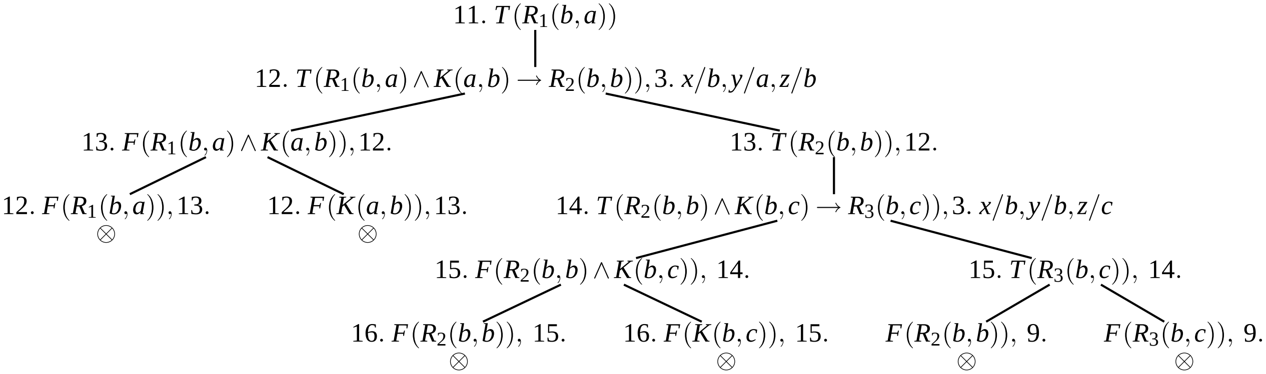
$$\begin{gathered}\forall x R_0(x,x)\\ \forall x\forall y\forall z(R_0(x,y)\land K(y,z)\to R_1(x,z))\\ \forall x\forall y\forall z(R_1(x,y)\land K(y,z)\to R_2(x,z))\\ \vdots\\ \forall x\forall y\forall z(R_{k-1}(x,y)\land K(y,z)\to R_k(x,z))\end{gathered}$$

The graph can be represented as $K(a,b)$, $K(b,a)$ and $K(b,c)$.

b)



The subtree from node 11 continues in the next page.



The finished tableau is contradictory and the claim holds.

Solution to Problem 5

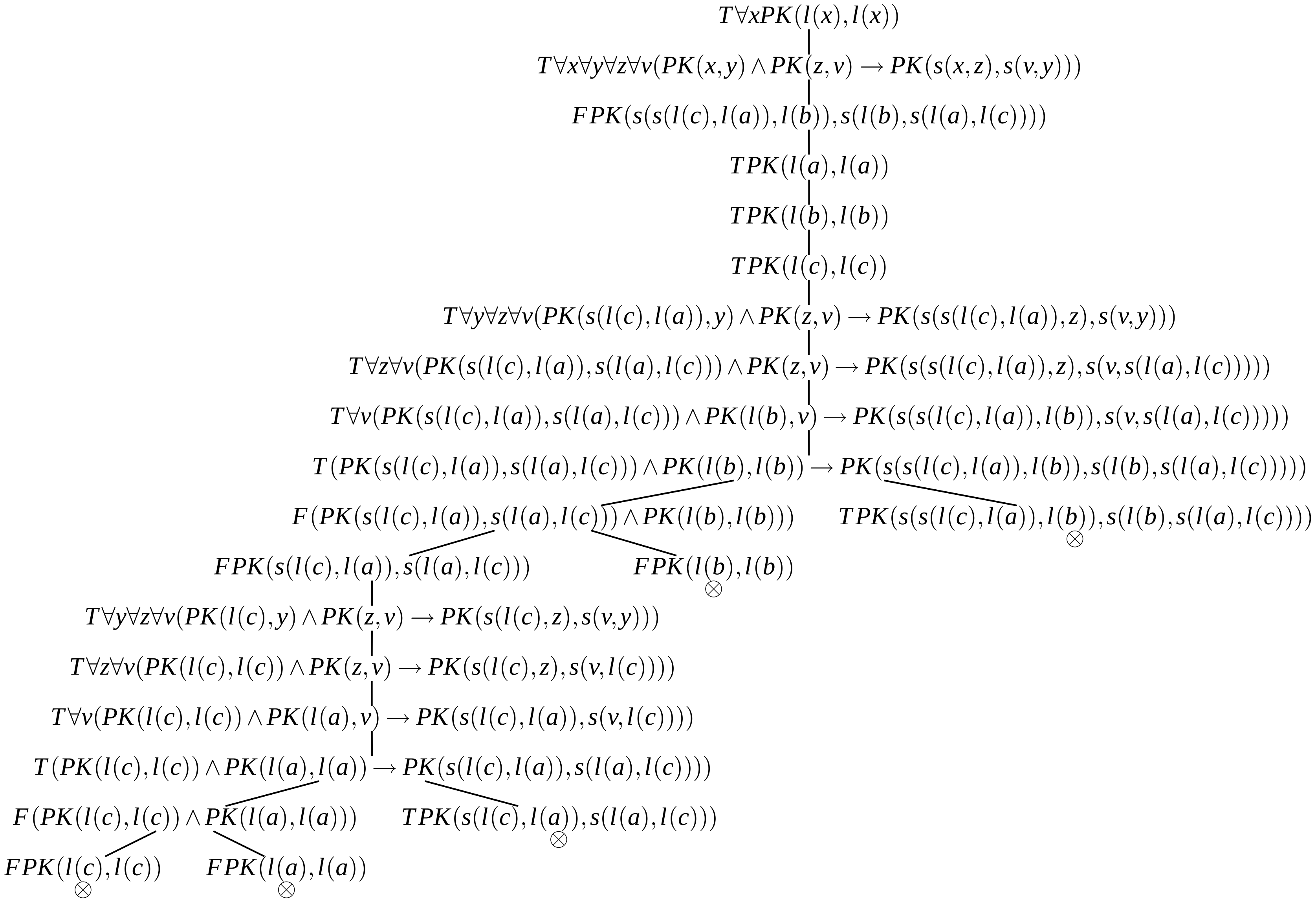
Define predicate PK as follows:

$$\begin{gathered}\forall x PK(l(x), l(x)) \\ \forall x \forall y \forall v \forall w (PK(x,v) \wedge PK(y,w) \rightarrow PK(s(x,y), s(w,v)))\end{gathered}$$

We show that

$$PK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$$

is a logical consequence of the definition of PK with the semantic tableau given in the next page.


$T\forall x PK(l(x),l(x))$
$T\forall x\forall y\forall z\forall v(PK(x,y)\wedge PK(z,v)\rightarrow PK(s(x,z),s(v,y)))$
$FPK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$
$TPK(l(a),l(a))$
$TPK(l(b),l(b))$
$TPK(l(c),l(c))$
$T\forall y\forall z\forall v(PK(s(l(c),l(a)),y)\wedge PK(z,v)\rightarrow PK(s(s(l(c),l(a)),z),s(v,y)))$
$T\forall z\forall v(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(z,v)\rightarrow PK(s(s(l(c),l(a)),z),s(v,s(l(a),l(c)))))$
$T\forall v(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),v)\rightarrow PK(s(s(l(c),l(a)),l(b)),s(v,s(l(a),l(c)))))$
$T(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),l(b))\rightarrow PK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c)))))$
$F(PK(s(l(c),l(a)),s(l(a),l(c)))\wedge PK(l(b),l(b)))$
$TPK(s(s(l(c),l(a)),l(b)),s(l(b),s(l(a),l(c))))$
$\otimes$
$FPK(s(l(c),l(a)),s(l(a),l(c)))$
$FPK(l(b),l(b))$
$\otimes$
$T\forall y\forall z\forall v(PK(l(c),y)\wedge PK(z,v)\rightarrow PK(s(l(c),z),s(v,y)))$
$T\forall z\forall v(PK(l(c),l(c))\wedge PK(z,v)\rightarrow PK(s(l(c),z),s(v,l(c))))$
$T\forall v(PK(l(c),l(c))\wedge PK(l(a),v)\rightarrow PK(s(l(c),l(a)),s(v,l(c))))$
$T(PK(l(c),l(c))\wedge PK(l(a),l(a))\rightarrow PK(s(l(c),l(a)),s(l(a),l(c))))$
$F(PK(l(c),l(c))\wedge PK(l(a),l(a)))$
$TPK(s(l(c),l(a)),s(l(a),l(c)))$
$\otimes$
$FPK(l(c),l(c))$
$\otimes$
$FPK(l(a),l(a))$
$\otimes$

Solution to Problem 6

Let predicate $K(x)$ denote that x is Father Christmas and predicate $J(x)$ denote that x is Santa Claus. Thus we get the following sentences:

1. $\exists x K(x) \wedge \forall x \forall y (K(x) \wedge K(y) \rightarrow x = y)$,
2. $\forall x (J(x) \rightarrow K(x))$, ja
3. $\forall x (K(x) \rightarrow J(x))$.

Sentence 4 is of the form: $\exists x J(x) \wedge \forall x \forall y (J(x) \wedge J(y) \rightarrow x = y)$. The semantic tableau: