T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 1 ([Nerode and Shore, 1997], Chapter I, Sections 1 and 2)
January 30–February 1, 2008

Solutions to demonstration problems

Solution to Problem 4

a) Yes, an atomic proposition.

b) No, does not contain even number of parentheses.

c) Yes, give for example a parse tree for the proposition.

d) No, natural language.

Solution to Problem 5

a) $\neg A \rightarrow \neg B$, when
A = "You help"
B = "I can finish my work"

b) $A \vee B \vee C$, when
A = "I walk to work"
B = "I ride a bicycle to work"
C = "Sometimes I drive a car to work"

c) Either: A, when
A = "Merja and Arto are coming to visit us"
or: $A \wedge B$, when
A = "Merja is coming to visit us"
B = "Arto is coming to visit us"

d) For example: $A \rightarrow \neg B$ or $A \wedge \neg B$, when
A = "You have been naughty"
B = "You will get dessert"

e) For example: $A \wedge B$, when
A = "The manual was long"
B = "The manual was not long enough"

f) $A \vee \neg A \rightarrow \neg B \vee (C \wedge D)$, when
A = “Somebody asks me”
B = “He should buy a car”
C = “He should live far from his workplace”
D = “The price of benzine should get cheaper”

Solution to Problem 6
Using the common definitions for connective precedence:

a) $A \rightarrow (B \wedge C) \vee D$

b) $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$

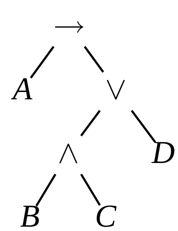
c) $(A \wedge (B \vee C)) \vee (A \wedge (C \vee D))$

d) $\neg(A \wedge B) \leftrightarrow (B \rightarrow C) \wedge A$

e) $\neg A \wedge \neg B \rightarrow \neg(A \vee B)$

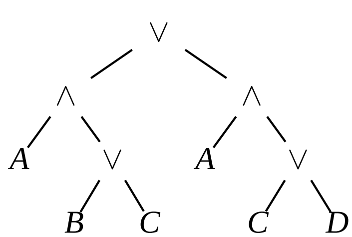
Solution to Problem 7
The form of a statement is obtained from its outermost connective:

a) Implication.



b) Implication.

c) Disjunction.



d) Equivalence.

e) Implication.

Solution to Problem 8
$\neg A \rightarrow (\neg B \rightarrow C)$, $\neg(\neg A \rightarrow B) \rightarrow C$, $\neg A$, $\neg B \rightarrow C$, $\neg(\neg A \rightarrow B)$, C, A, $\neg B$, $\neg A \rightarrow B$, B. In addition the propositional statement is its own component.

Solution to Problem 9
Basic case: A set of 0 elements (assuming $0 \in \mathbb{N}$), that is, the empty set, has one subset, itself. In addition $2^0 = 1$.
Induction hypothesis: We assume that the claim holds for $n = k$, that is, a set of k elements has 2^k subsets.
Induction step: Consider an arbitrary set A that has $k+1$ elements. Choose arbitrary $a \in A$. The subsets of A divide into two cases: the ones that contain a and the ones that don't contain a. Let B be the set of subsets of A containing a and C be the set of subsets of A not containing a. The set C is now the set of subsets of a k element set (consider why!) and by induction hypothesis $|C| = 2^k$. On the other hand, each element in B (that is, a each subset of A containing a) can be bijectively mapped into an element of C by removing a. Thus $|B| = |C|$. Since each subset of A belongs to either B or C (but not both!) the number of subsets of A is $|B| + |C| = 2^n + 2^n = 2 * 2^n = 2^{n+1}$. $\square$

Solution to Problem 10
We prove the claim using induction on the length of proposition
Basic case: A proposition of the length 1 is a propositional symbol and it contains 0 parentheses (0 is an even number).
Induction hypothesis: If the length of the proposition is less than n, then it contains an even number of parenthesis.
Induction step: Let the length of the proposition to be n ($n > 1$). Then the proposition is of the form $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$ or $(\alpha \leftrightarrow \beta)$. Here α and β are some propositions that are shorter than n. By induction hypothesis they contain an even number of parenthesis and so the proposition itself contains also an even number of parenthesis. This completes the proof by induction.