T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 8 ([NS, 1997], Predicate Logic, Chapters 4 and 9)
April 2–4, 2008

Tutorial problems

1. Define predicate $Y(x,y)$ (there is a connection from city x to city y) using the predicate $L(x,y)$ (there is a flight from city x to city y).

2. Show that the following sentences are not valid by constructing a structure in which the sentence is false, i.e., construct a counter-example.

   a) $\forall x(P(x) \to R(x)) \land \forall x(Q(x) \to R(x)) \to \forall x(P(x) \to Q(x))$

   b) $\forall x \forall y(R(x,y) \to R(y,x)) \to \exists y \forall x R(x,y)$

3. Transform the following sentences into clausal form.

   a) $\neg(\exists x A(x) \lor \exists x B(x) \to \exists x(A(x) \lor B(x)))$

   b) $(\forall x P(x) \to \exists x \forall y Q(x,y)) \to \neg \forall y P(y))$

Demonstration problems

4. Let R be a binary predicate with interpretation $R^{\mathcal{S}} \subseteq U \times U$ (the set U is the domain of structure $\mathcal{S}$). In the following table we give definitions for some properties of relation $R^{\mathcal{S}}$.

Property	Definition
reflexivity	$\forall x R(x,x)$
irreflexivity	$\forall x \neg R(x,x)$
symmetry	$\forall x \forall y(R(x,y) \to R(y,x))$
asymmetry	$\forall x \forall y(R(x,y) \to \neg R(y,x))$
transitivity	$\forall x \forall y \forall z(R(x,y) \land R(y,z) \to R(x,z))$
seriality	$\forall x \exists y R(x,y)$

   Consider a domain U consisting of people. Give examples of relations $R^{\mathcal{S}}$, ($\emptyset \subset R^{\mathcal{S}} \subset U^2$), that have properties described above.

5. Show that the following sentences are not valid by constructing a structure in which the sentence is false, i.e., construct a counter-example.

   a) $\forall x \exists y P(x,y) \to \exists y \forall x P(x,y)$

b) $\exists x(P(x) \vee Q(x)) \rightarrow \exists x P(x) \wedge \exists x Q(x)$

c) $\neg\forall x(P(x) \rightarrow R(x)) \vee \neg\forall x(P(x) \rightarrow \neg R(x))$

6. Transform the following sentences into conjunctive normal form and perform skolemization.

a) $\forall y(\exists x P(x,y) \rightarrow \forall z Q(y,z)) \wedge \exists y(\forall x R(x,y) \vee \forall x Q(x,y))$

b) $\exists x \forall y R(x,y) \leftrightarrow \forall y \exists x P(x,y)$

c) $\forall x \exists y Q(x,y) \vee (\exists x \forall y P(x,y) \wedge \neg\exists x \exists y P(x,y))$

d) $\neg(\forall x \exists y P(x,y) \rightarrow \exists x \exists y R(x,y)) \wedge \forall x \neg\exists y Q(x,y)$

7. Use the rules in Lemma 9.1 [NS, 1997, page 129] to obtain rules for the following cases.

a) $\forall x \phi(x) \rightarrow \psi$

b) $\exists x \phi(x) \rightarrow \psi$

c) $\phi \rightarrow \forall x \psi(x)$

d) $\phi \rightarrow \exists x \psi(x)$

8. Transform the following sentences into clausal form.

a) $\neg\exists x((P(x) \rightarrow P(a)) \wedge (P(x) \rightarrow P(b)))$

b) $\forall y \exists x P(x,y)$

c) $\neg\forall y \exists x G(x,y)$

d) $\exists x \forall y \exists z(P(x,z) \vee P(z,y) \rightarrow G(x,y))$