Tutorial problems

1. Find disjunctive and conjunctive normal forms for the following propositions using the transformation rules.
   a) \( \neg((\neg(A \rightarrow B) \land (\neg B \leftrightarrow C)) \)
   b) \( A \lor B \rightarrow (\neg A \rightarrow \neg B) \)

2. Find disjunctive and conjunctive normal forms for the following propositions using semantic tableaux.
   a) \( (C \rightarrow A) \rightarrow B \)
   b) \( (B \rightarrow \neg A) \rightarrow ((B \lor \neg A) \rightarrow \neg B) \)

3. Find the clause form for \( \neg A \lor (B \rightarrow (C \leftrightarrow B)) \). Give a truth assignment \( A \) such that it is a model for the set of clauses.

Demonstration problems

4. Find disjunctive and conjunctive normal forms for the following propositions using (1) the transformation rules and (2) semantic tableaux.
   a) \( A \rightarrow (B \rightarrow C) \)
   b) \( \neg A \leftrightarrow ((A \lor \neg B) \rightarrow B) \)
   c) \( \neg((A \leftrightarrow \neg B) \rightarrow C) \)
   d) \( P_1 \land P_2 \leftrightarrow (P_1 \rightarrow P_2) \lor (P_2 \rightarrow P_3) \)

5. Use semantic tableaux to prove that the rules used to find CNF/DNF of a proposition maintain logical equivalence.

6. Find CNFs for the following propositions both by applying the transformation rules and using semantic tableaux.
   a) \( (P \land \neg P) \lor (Q \land \neg Q) \)
   b) \( (P_1 \land \neg P_1) \lor \cdots \lor (P_n \land \neg P_n) \)

Use semantic tableaux to prove that CNF obtained for a) is unsatisfiable.
7. Find a clause form for

$$(A \rightarrow ((A \rightarrow A) \rightarrow A)) \rightarrow ((A \rightarrow (A \rightarrow A)) \rightarrow (A \rightarrow A)).$$

8. Consider the set of clauses:

$$S = \{ \{ A_0, A_1 \}, \{ \neg A_0, \neg A_1 \}, \{ A_1, A_2 \}, \{ \neg A_1, \neg A_2 \}, \ldots, \{ A_{n-1}, A_n \}, \{ \neg A_{n-1}, \neg A_n \}, \{ A_n, A_0 \}, \{ \neg A_n, \neg A_0 \} \}$$

Give truth assignment $\mathcal{A}$ such that $\mathcal{A} | = S$.

9. Horn-clause is a clause that has exactly one positive literal. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be models for a set of Horn-clauses $S$. Show that also $\mathcal{A} = \mathcal{A}_1 \cap \mathcal{A}_2$ is a model of $S$. 