T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 5 ([NS, 1997] Chapters 4 and 8)
February 27–29, 2008

Tutorial problems

1. Find disjunctive and conjunctive normal forms for the following propositions using the transformation rules.

a) $\neg(\neg(A \rightarrow B) \wedge (\neg B \leftrightarrow C))$

b) $A \vee B \rightarrow (\neg A \rightarrow \neg B)$

2. Find disjunctive and conjunctive normal forms for the following propositions using semantic tableaux.

a) $(C \rightarrow A) \rightarrow B$

b) $(B \rightarrow \neg A) \rightarrow ((B \vee \neg A) \rightarrow \neg B)$

3. Find the clause form for $\neg A \vee (B \rightarrow \neg(C \leftrightarrow B))$. Give a truth assignment $\mathcal{A}$ such that it is a model for the set of clauses.

Demonstration problems

4. Find disjunctive and conjunctive normal forms for the following propositions using (1) the transformation rules and (2) semantic tableaux.

a) $A \rightarrow (B \rightarrow C)$

b) $\neg A \leftrightarrow ((A \vee \neg B) \rightarrow B)$

c) $\neg((A \leftrightarrow \neg B) \rightarrow C)$

d) $P_1 \wedge P_2 \leftrightarrow (P_1 \rightarrow P_2) \vee (P_2 \rightarrow P_3)$

5. Use semantic tableaux to prove that the rules used to find CNF/DNF of a proposition maintain logical equivalence.

6. Find CNFs for the following propositions both by applying the transformation rules and using semantic tableaux.

a) $(P \wedge \neg P) \vee (Q \wedge \neg Q)$

b) $(P_1 \wedge \neg P_1) \vee \cdots \vee (P_n \wedge \neg P_n)$

Use semantic tableaux to prove that CNF obtained for a) is unsatisfiable.

7. Find a clause form for

$$(A \to ((A \to A) \to A)) \to ((A \to (A \to A)) \to (A \to A)).$$

8. Consider the set of clauses:

$$\begin{aligned} S \;=\; & \{\{A_0, A_1\}, \{\neg A_0, \neg A_1\}, \{A_1, A_2\}, \{\neg A_1, \neg A_2\}, \ldots, \\ & \{A_{n-1}, A_n\}, \{\neg A_{n-1}, \neg A_n\}, \{A_n, A_0\}, \{\neg A_n, \neg A_0\}\} \end{aligned}$$

Give truth assignment $\mathcal{A}$ such that $\mathcal{A} \models S$.

9. Horn-clause is a clause that has exactly one positive literal. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be models for a set of Horn-clauses S. Show that also $\mathcal{A} = \mathcal{A}_1 \cap \mathcal{A}_2$ is a model of S.