T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 4 ([NS 1997], Chapter I, Sections 4 and 7)
February 20–22, 2008

Tutorial problems

1. Use semantic tableaux to prove the following:

a) $\models C \vee (\neg A \vee (B \rightarrow (\neg C \leftrightarrow B)))$

b) $\{A \vee (B \wedge C)\} \models (A \vee B) \wedge (A \vee C)$

c) $\{C \rightarrow \neg A, B \rightarrow (A \vee C)\} \models (C \rightarrow A) \rightarrow \neg C$

2. Use a semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a) $\{\neg A \wedge \neg B \leftrightarrow C \vee D, \neg(\neg C \rightarrow D\} \models A \wedge B$

b) $\models (P \vee Q \vee \neg R) \wedge ((\neg R \vee Q \vee P) \rightarrow (R \vee Q) \wedge \neg Q \wedge \neg P)$

3. Give a Hilbert style proof for

$$\{B \rightarrow A, \neg A\} \vdash \neg B.$$

Demonstration problems

4. Peirce arrow is defined as:

$$A \downarrow B \Leftrightarrow_{def} \neg A \wedge \neg B.$$

Define semantic tableaux rules for it.

5. Use semantic tableux to show that the following propositions are valid.

a) $A \rightarrow (B \rightarrow B)$,

b) $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C)$,

c) $(A \rightarrow B) \wedge (A \rightarrow C) \rightarrow (A \rightarrow B \wedge C)$ and

d) $(A \rightarrow C) \wedge (B \rightarrow C) \wedge (A \vee B) \rightarrow C$.

6. Use semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a) $\{B \rightarrow A, C \rightarrow B, (C \rightarrow A) \rightarrow D\} \models D$

b) $\{A \to C, A \vee B, \neg D \to \neg B\} \models C \to D$

c) $\models (A \to (B \to C)) \to ((A \to C) \to (A \to B))$

d) $\models (\neg B \to (A \to C)) \to (A \to (B \vee C))$

7. Recall the specification for two traffic light posts positioned in the intersection of two one-way streets discussed earlier in tutorials. Use semantic tableaux to prove that "the red lights can't be on at the same" is a logical consequence of the set of propositions describing the behaviour of the system.

8. Use the proof system by Hilbert to prove the following.

a) $\vdash P \to P$

b) $\{P \to Q, Q \to R\} \vdash P \to R$

c) $\{P, Q \to (P \to R)\} \vdash Q \to R$