

**Tutorial problems**

1. Use semantic tableaux to prove the following:

a)  $\models C \vee (\neg A \vee (B \rightarrow (\neg C \leftrightarrow B)))$

b)  $\{A \vee (B \wedge C)\} \models (A \vee B) \wedge (A \vee C)$

c)  $\{C \rightarrow \neg A, B \rightarrow (A \vee C)\} \models (C \rightarrow A) \rightarrow \neg C$

2. Use a semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a)  $\{\neg A \wedge \neg B \leftrightarrow C \vee D, \neg(\neg C \rightarrow D)\} \models A \wedge B$

b)  $\models (P \vee Q \vee \neg R) \wedge ((\neg R \vee Q \vee P) \rightarrow (R \vee Q) \wedge \neg Q \wedge \neg P)$

3. Give a Hilbert style proof for

$$\{B \rightarrow A, \neg A\} \vdash \neg B.$$

**Demonstration problems**

4. Peirce arrow is defined as:

$$A \downarrow B \Leftrightarrow_{def} \neg A \wedge \neg B.$$

Define semantic tableaux rules for it.

5. Use semantic tableaux to show that the following propositions are valid.

a)  $A \rightarrow (B \rightarrow B),$

b)  $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow (A \rightarrow C),$

c)  $(A \rightarrow B) \wedge (A \rightarrow C) \rightarrow (A \rightarrow B \wedge C)$  and

d)  $(A \rightarrow C) \wedge (B \rightarrow C) \wedge (A \vee B) \rightarrow C.$

6. Use semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a)  $\{B \rightarrow A, C \rightarrow B, (C \rightarrow A) \rightarrow D\} \models D$

- b)  $\{A \rightarrow C, A \vee B, \neg D \rightarrow \neg B\} \models C \rightarrow D$
- c)  $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$
- d)  $\models (\neg B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \vee C))$

7. Recall the specification for two traffic light posts positioned in the intersection of two one-way streets discussed earlier in tutorials. Use semantic tableaux to prove that “the red lights can’t be on at the same” is a logical consequence of the set of propositions describing the behaviour of the system.

8. Use the proof system by Hilbert to prove the following.

- a)  $\vdash P \rightarrow P$
- b)  $\{P \rightarrow Q, Q \rightarrow R\} \vdash P \rightarrow R$
- c)  $\{P, Q \rightarrow (P \rightarrow R)\} \vdash Q \rightarrow R$