T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 3 ([NS 1997], Chapter I, Section 3)
February 13–15, 2008

Tutorial problems

1. Let Σ be a set of propositional statements and ϕ a propositional statement.

a) Show that if $\Sigma \models \neg\phi$ then $\Sigma \cup \{\phi\}$ is unsatisfiable.

b) Show that the following claim **doesn't hold**: if $\Sigma \not\models \phi$ then $\Sigma \models \neg\phi$.

2. Formalize the following text using propositional logic:

"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is immortal or a mammal, then it is horned. The unicorn is magical if it is horned." [Russell and Norvig: Artificial Intelligence: Modern Approach]

3. Let Σ be a set of propositional statements and ϕ and ψ propositional statements. Show that if $\phi \in \Sigma$ and $\phi \equiv \psi$, then $(\Sigma \setminus \{\phi\}) \cup \{\psi\} \equiv \Sigma$.

Demonstration problems

4. Prove the following claims.

a) If $\Sigma \models \phi$ and $\Sigma \models \neg\phi$ for some ϕ, then the set of propositions Σ is unsatisfiable.

b) If set of propositions Σ has exactly one model, then for all propositions ϕ either $\Sigma \models \phi$ or $\Sigma \models \neg\phi$ (but not both).

5. Proof the following properties of logical consequences.

a) $\Sigma \subseteq \mathrm{Cn}(\Sigma)$.

b) Monotonicity: $\Sigma_1 \subseteq \Sigma_2 \Rightarrow \mathrm{Cn}(\Sigma_1) \subseteq \mathrm{Cn}(\Sigma_2)$.

c) $\Sigma \models \phi \Rightarrow \mathrm{Cn}(\Sigma) = \mathrm{Cn}(\Sigma \cup \{\phi\})$.

d) $\mathrm{Cn}(\mathrm{Cn}(\Sigma)) = \mathrm{Cn}(\Sigma)$.

6. Use propositional logic to formalize a voting system for three voters, for which the models give the positive or negative voting result. How does the system change if there are four voters, and the vote of the chair decides in case of a tie?

7. The reader of the Helsinki area travel card has three lights which are lit according to the following principles (see `http://www.ytv.fi/ENG/transport/guide/travel_card/` for more information).

1. Green light: a valid period ticket / value ticket / transfer ticket.
2. Green and yellow light: less than or equal to 3 full days' valid period / less than or equal to 5 euros' value on the travel card.
3. Red light: period / transfer not valid or other error.

Formalize the system using propositional logic and find out what kind of models the set of propositions has.