T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 12 ([Huth & Ryan, 2000], Chapter 4)
Apr 29–30 and May 2, 2008

Tutorial problems

1. (a) Write down a program P such that $\models_p$ [true] P [y==x+2] holds and prove that this is so.

(b) Write down a program P such that P contains an if-statement and

$$\models_p [\texttt{true}]\ P\ [\texttt{z>x+y+4}]$$

holds, and prove that this is so.

2. Show that the following holds for program Prog:

$$\models_p [\texttt{true}]\ \texttt{Prog}\ [\texttt{x==v-z}],$$

where Prog is as follows:

```
x=0;
y=0;
while(!(y == z)) {
     y=y+1;
     x=x-1
}
x=x+v;
```

3. (a) For any B_1, B_2 and P explain why $\models_p [B_1]\ P\ [B_2]$, whenever the relation $\models_t [B_1]\ P\ [B_2]$ holds.

(b) Show that the following holds for program Prog in Exercise 2:

$$\models_t [\texttt{z>=0}]\ \texttt{Prog}\ [\texttt{x==v-z}].$$

Demonstration problems

4. Use propositional logic to prove the equivalence of the following statements.

(a) !(a==b || a<b)

(b) `a!=b&&!(b>a)`

5. Prove the partial correctness in the following cases.

(a) $\models_p$ [`x>0`] `y=x+1` [`y>1`]

(b) $\models_p$ [`true`] `y=x; y=x+x+y` [`y==3*x`]

(c) $\models_p$ [`x>1`] `a=1; y=x; y=y-a` [`y>0&&x>y`]

6. Show that $\models_p$ [`true`] `P` [`z==` min `(x,y)`], where `P` is the following program:

```
if(x>y) then{
     z=y
}else{
     z=x
}
```

7. Show that

(a) $\models_p$ [`true`] `Sum` [`z==x+y`]

(b) $\models_t$ [`0<=y`] `Sum` [`z==x+y`]

where `Sum` is the following program:

```
z=x;
v=y;
while(!(v==0)){
     z=z+1;
     v=v-1
}
```