Tutorial problems

1. (a) Write down a program $P$ such that $|=p \ [\text{true}] \ P \ [y=x+2]$ holds and prove that this is so.
   
   (b) Write down a program $P$ such that $P$ contains an if-statement and
   
   $$|=p \ [\text{true}] \ P \ [z>x+y+4]$$
   
   holds, and prove that this is so.

2. Show that the following holds for program $\text{Prog}$:

   $$|=p \ [\text{true}] \ \text{Prog} \ [x=v-z],$$

   where $\text{Prog}$ is as follows:

   
   ```
   x = 0;
y = 0;
while(!(y == z)) {
    y = y + 1;
x = x - 1
}
x = x + v;
   ```

3. (a) For any $B_1$, $B_2$ and $P$ explain why $|=p \ [B_1] \ P \ [B_2]$, whenever the relation $|=t \ [B_1] \ P \ [B_2]$ holds.
   
   (b) Show that the following holds for program $\text{Prog}$ in Exercise 2:

   $$|=t \ [z>=0] \ \text{Prog} \ [x=v-z].$$

Demonstration problems

4. Use propositional logic to prove the equivalence of the following statements.

   (a) $!(a==b \ | \ a<b)$
(b) \( a != b && b > a \)

5. Prove the partial correctness in the following cases.

(a) \( \models_p [x > 0] \ y = x + 1 \ [y > 1] \)
(b) \( \models_p [true] \ y = x ; y = x + x + y \ [y == 3 \times x] \)
(c) \( \models_p [x > 1] \ a = 1 ; y = x ; y = y - a \ [y > 0 && x > y] \)

6. Show that \( \models_p [true] \ P \ [z == min (x,y)] \), where \( P \) is the following program:

\[
\text{if}(x > y) \text{ then } \{ \\
\quad z = y \\
\} \text{ else } \{ \\
\quad z = x \\
\}
\]

7. Show that

(a) \( \models_p [true] \ Sum \ [z == x + y] \)
(b) \( \models_r [0 <= y] \ Sum \ [z == x + y] \)

where \( Sum \) is the following program:

\[
z = x ; \\
v = y ; \\
\text{while}(! (v == 0)) \{ \\
\quad z = z + 1 ; \\
\quad v = v - 1 \\
\}
\]