T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 11 ([NS, 1997], Predicate Logic, Chapters 10 – 14)
April 23–25, 2008

Tutorial problems

1. (a) Compute $\sigma\lambda$ for substitutions $\sigma = \{x/g(y), y/h(z,w), z/a, w/x\}$ and $\lambda = \{x/w, y/f(a,b), z/b\}$.

(b) Find the most general unifier of

$$\{Q(h(x,y),w), Q(h(g(v),a), f(v)), Q(h(g(v),a), f(b))\}$$

(c) Explain why none of the following sets of literals is unifiable:

$$\{P(x,a), P(b,c)\},\ \{P(x,a), P(f(x),y)\},\text{ and } \{P(f(x),z), P(a,w)\}.$$

2. Find resolvents for each of the following.

(a) $\{P(x,y), P(y,z)\}$, $\{\neg P(u, f(u))\}$

(b) $\{P(x,x), \neg R(x, f(x))\}$, $\{R(x,y), Q(y,z)\}$

(c) $\{P(x,y), \neg P(x,x), Q(x, f(x), z)\}$, $\{\neg Q(f(x), x, z), P(x,z)\}$

3. We know that:

1) If a brick is on another brick, then it is not on the table.

2) Every brick is either on the table or on another brick.

3) No brick is on a brick which is also on some other brick.

Use resolution to prove that if a brick is on another brick, the other brick is on the table.

Demonstration problems

4. Define the Herbrand universe and Herbrand base for the following sets of clauses.

a) $\{\{\neg G(x,c)\}\}$,

b) $\{\{P(f(y),y)\}\}$,

c) $\{\{P(x)\}, \{\neg P(a), \neg P(b)\}\}$,

d) $\{\{\neg P(x,y), \neg P(y,z), G(x,z)\}\}$,

e) $\{\{\neg P(x,y)\},\ \{Q(a,x),\ Q(b,f(y))\}\}$, ja

f) $\{\{P(x),\ Q(f(x,y))\}\}$

5. Consider

$$\Sigma = \{\forall x P(x,a,x),\ \neg\exists x\exists y\exists z(P(x,y,z) \wedge \neg P(x,f(y),f(z)))\}.$$

a) Transform Σ into a set of clauses S.

b) Define the Herbrand universe H and Herbrand base B of S.

c) Let Herbrand structures be subsets of the Herbrand base. Find the subset minimal and maximal Herbrand models of S.

6. Transform the problem of deciding the validity of sentence

$$\exists x\exists y(P(x) \leftrightarrow \neg P(y)) \rightarrow \exists x\exists y(\neg P(x) \wedge P(y))$$

into the problem of satisfiability of a propositional logic statement and solve the problem.

7. Find the composition of substitutions $\{x/y, y/b, z/f(x)\}$ and $\{x/g(a), y/x, w/c\}$.

8. Find the most general unifiers for the following sets of literals.

a) $\{P(x,g(y),f(a)), P(f(y),g(f(z)),z)\}$

b) $\{P(x,f(x),g(y)), P(a,f(g(a)),g(a)), P(y,f(y),g(a))\}$

c) $\{P(x,f(x,y)), P(y,f(y,a)), P(b,f(b,a))\}$

d) $\{P(f(a),y,z), P(y,f(a),b), P(x,y,f(z))\}$

9. Show that

a) the composition of substitutions is not commutative, that is, there are substitutions σ and λ such that $\sigma\lambda \neq \lambda\sigma$.

b) a most general unifier is not unique, that is, there is a set of literals S such that it has two most general unifiers σ and λ such that $\sigma \neq \lambda$.

10. Unify $\{P(x,y,z), P(f(w,w), f(x,x), f(y,y))\}$.

11. Use resolution to prove that there are no barbers, when

a) all barbers shave everyone who does not shave himself, and

b) no barber shaves anyone who shaves himself.

12. We use groud terms 0, $s(0)$, $s(s(0))$, ..., to represent natural numbers 0, 1, 2, ..., where 0 is a constants and s is a unary function such that $s(x) = x+1$ for all natural numbers x.

a) Let predicates $J2(x), J3(x)$ and $J6(x)$ represent that a natural number x is divisible by two, three and six, respectively. Define these predicates with sentences in predicate logic using the definitions of $J2$ and $J3$ to define $J6$.

b) Use resolution to prove that if a natural number x is divisible by two and three, then natural number $x+6$ is divisible by six.