**Tutorial problems**

1. Use semantic tableaux to prove that the last sentence is not a logical consequence of the first two sentences.

   "Penguins are black and white. Some old tv shows are black and white. Therefore, some penguins are old tv shows."

2. Let $R$ be a binary predicate with interpretation $R^{\mathcal{S}} \subseteq U \times U$ (the set $U$ is the domain of structure $\mathcal{S}$). In the following table we give definitions for some properties of relation $R^{\mathcal{S}}$.

   | Property | Definition |
   |----------|------------|
   | reflexivity | $\forall x R(x,x)$ |
   | irreflexivity | $\forall x \neg R(x,x)$ |
   | symmetry | $\forall x \forall y (R(x,y) \rightarrow R(y,x))$ |
   | asymmetry | $\forall x \forall y (R(x,y) \rightarrow \neg R(y,x))$ |
   | transitivity | $\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$ |
   | seriality | $\forall x \exists y R(x,y)$ |

   Use semantic tableaux to show that an irreflexive and transitive relation is also asymmetric.

3. Formalize the following sentences using predicate logic and use semantic tableaux to prove that sentence 4 is a logical consequence of sentences 1-3.

   1. Alders are broad-leaved trees.
   2. Trees are alders, spruces or pines.
   3. Spruces and pines are coniferous trees.
   4. Trees are coniferous or broad-leaved.

**Demonstration problems**

4. A *directed* graph consists of a set of nodels and a set of *directed* egdes between the nodes. Assume that nodes a represented with constants $\{a, b, \ldots\}$ and edges with a binary predicate $K(x,y) = $ "there is an edge from node $x$ to node $y$".

1. Define predicates $R_n(x,y)$ = "node $y$ is reachable from node $x$ using $n$ edges", for $n = 0, 1, 2, \ldots, k$. Represent the following graph with predicate $K$.

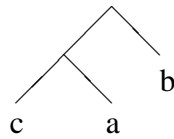$$a \underset{\longleftarrow}{\overset{\longrightarrow}{\phantom{xx}}} b \longrightarrow c$$
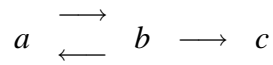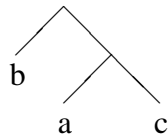
2. Use semantic tableaux to show that

$$\exists x(R_2(x,x) \wedge R_3(x,c))$$

is a logical consequence of the representation of the graph and definitions of predicates $R_2$ and $R_3$

5. We represent binary trees using a binary function $s$ (internal nodes) and a unary function $l$ (leaf nodes). Thus the representation of the upper tree is the term $s(s(l(c), l(a)), l(b))$.

a) Let predicate $PK(x,y)$ denote that binary tree $x$ is the mirror-image of binary tree $y$. Define predicate PK.

b) Use semantic tableaux to proof that the upper binary tree is the mirror-image of the lower binary tree.

6. Quantifier $\exists! x$ is used to denote "there is only one $x$". Sentence $\exists! x \phi(x)$ can be represented as

$$(\exists x \phi(x)) \wedge (\forall x \forall y(\phi(x) \wedge \phi(y) \rightarrow x = y)).$$

Formalize the following sentences using predicate logic:

1. There is only one Father Christmas.
2. Every Santa Claus is Father Christmas.
3. Every Father Christmas is Santa Claus.
4. There is only one Santa Clause.

Use semantic tableaux to prove that sentence 4 is a logical consequence of sentences 1-3.