Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)
(a) Define the following concepts: model, ground term, and unifier. (3 × 2p)
(b) What is meant by the notation $\phi \equiv \psi$?
Prove in detail that if $\phi \equiv \psi$ and $\psi \equiv \chi$, then $\phi \equiv \chi$. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:
(a) $\models (A \to B) \land (B \to C) \land (C \to A) \to (\neg B \to \neg C)$
(b) $\models \forall y \forall x (R(x, y) \to P(x)) \to \forall x (\exists y R(x, y) \to P(x))$
Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses $S$) for the sentence
$\neg (\forall x \exists y (P(x) \lor Q(y))) \to \exists y \forall x (P(x) \lor Q(y))$.
Make $S$ as simple as possible. Prove that $S$ is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings "", "a", "b", "aa", "ab", "ba", "bb", . . . that consist of letters a ja b using ground terms $e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), . . . ,$
built of a constant symbol $e$, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter $a$ or $b$ at the beginning of a string $x$. Thus $a(b(e))$ is interpreted as $a(b(\"\")) = a(\"b\") = \"ab\"$.
(a) Define predicate $P(x) = \"the letter a occurs an even number of times in the string x\"$ using predicate logic so that your definition covers all finite strings represented as explained above.
(b) Give a model $S \models \Sigma$ of your definition $\Sigma$ on the basis of which it holds that
$\Sigma \not\models P(b(a(b(e))))$.

Assignment 5 (10p)
Explain how the weakest precondition $B_1$ of an if-statement
\[ \text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\} \]
can be formed given a postcondition $B_2$ for it.
Consider the following program Minus:
\[ v = x ; z = y ; \text{while}(! (z == 0)) \{ z = z - 1 ; v = v - 1 \} . \]
Use weakest preconditions and a suitable invariant to establish
$\models_p [\text{true}] \text{ Minus } [v = x - y]$.