T-79.3001 Logic in Computer Science: Foundations

Examination, March 7, 2009

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *model*, *ground term*, and *unifier*. ($3 \times 2p$)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that if $\phi \equiv \psi$ and $\psi \equiv \chi$, then $\phi \equiv \chi$. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \to B) \land (B \to C) \land (C \to A) \to (\neg B \to \neg C)$

(b) $\models \forall y \forall x (R(x,y) \to P(x)) \to \forall x (\exists y R(x,y) \to P(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\forall x \exists y (P(x) \lor Q(y)) \to \exists y \forall x (P(x) \lor Q(y))).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings "", "a", "b", "aa", "ab", "ba", "bb", ... that consist of letters a ja b using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol e, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(\text{""})) = a(\text{"}b\text{"}) = \text{"}ab\text{"}$.

(a) Define predicate $P(x) =$ "the letter a occurs an even number of times in the string x" using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models P(b(a(b(e)))).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=x;z=y;while(!(z==0)){z=z-1;v=v-1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.