Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

**Assignment 1** (10 p)

(a) Define the following concepts: an adequate set of connectives, Herbran universe, and total correctness. (3 \times 2 p)

(b) What is meant by the notation $\Sigma \models \phi$? Prove in detail that if $\Sigma \cup \{\phi\} \models \psi$, then $\Sigma \models \phi \rightarrow \psi$. (4 p)

**Assignment 2** (10 p) Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

(b) $\models \exists x(P(x) \lor Q(x)) \rightarrow \forall x(P(x) \lor Q(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** (10 p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses $S$) for the sentence

$$\neg(\forall x \exists y (P(x) \land Q(y)) \rightarrow \exists y \forall x (P(x) \land Q(y))).$$

Make $S$ as simple as possible. Prove that $S$ is unsatisfiable using resolution.

**Assignment 4** (10 p) Let us represent strings “”, “a”, “b”, “aa”, “ab”, “ba”, “bb”, … that consist of letters $a$ ja $b$ using ground terms

$$e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), \ldots,$$

built of a constant symbol $e$, which represents the empty string “”, and unary functions $a(x)$ and $b(x)$, that append the respective letter $a$ or $b$ at the beginning of a string $x$. Thus $a(b(e))$ is interpreted as $a(b(“”)) = a(“b”) = “ab”$.

(a) Define predicate $AB(x) = “$the string $x$ is of the form $abab\ldots ab$ where the string $ab$ repeats $n \geq 0$ times”$”$ using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $S \models \Sigma$ of your definition $\Sigma$ on the basis of which it holds that $\Sigma \not\models AB(b(a(e))).$

**Assignment 5** (10 p)

Explain how the weakest precondition $B_1$ of an if-statement

$$\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}$$

can be formed given a postcondition $B_2$ for it.

Consider the following program Minus:

$$v=x; z=y; \text{while}(! (z==0)) \{z=z-1; v=v-1\}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\text{true}] \text{ Minus } [v==x-y].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.