Assignment 1 (10p)
(a) Define the following concepts: formation tree, truth table, and unique names assumption. (3 × 2p)
(b) What is meant by the notation \( \models \phi \)?
Prove in detail that if \( \models \phi \rightarrow \psi \), then the set of sentences \( \Sigma = \{ \phi, \neg \psi \} \) is unsatisfiable. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:
(a) \( \models (B \rightarrow \neg A) \land (B \lor C) \land (C \rightarrow A) \rightarrow (A \leftarrow C) \)
(b) \( \{ \forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x) \} \not\models \forall y Q(y) \)
Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses \( S \)) for the sentence
\[ \neg \exists x (\exists y \neg R(x,y)) \rightarrow \exists z \neg R(z,x). \]
Make \( S \) as simple as possible. Prove that \( S \) is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings “”,” “a”, “b”, “aa”, “ab”, “ba”, “bb”,
...that consist of letters a ja b using ground terms
\[ e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), \ldots, \]
built of a constant symbol \( e \), which represents the empty string “”,” and unary functions \( a(x) \) and \( b(x) \), that append the respective letter a or b at the beginning of a string \( x \). Thus \( a(b(e)) \) is interpreted as \( a(b(“”)) = a(“b”) = “ab” \).

(a) Define predicate \( O(x) = “the number of occurences of a in the string x is odd” \) using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model \( S \models \Sigma \) of your definition \( \Sigma \) on the basis of which it holds that
\[ \Sigma \not\models O(a(b(a(e)))) \].

Assignment 5 (10p)
Explain how the weakest precondition \( B_1 \) of an if-statement
\[ \text{if}(B) \text{ then } \{ C_1 \} \text{ else } \{ C_2 \} \]
can be formed given a postcondition \( B_2 \) for it.
Consider the following program Minus:
\[ v = x; z = y; \text{while}(\neg(z == 0)) \{ z = z - 1; v = v - 1 \}. \]
Use weakest preconditions and a suitable invariant to establish
\[ \models_p \text{true] Minus [v==x-y}. \]

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.