T-79.3001 Logic in Computer Science: Foundations

Examination, October 29, 2008

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10*p*)

(a) Define the following concepts: *formation tree*, *truth table*, and *unique names assumption*. ($3 \times 2p$)

(b) What is meant by the notation $\models \phi$?

Prove in detail that if $\models \phi \rightarrow \psi$, then the set of sentences $\Sigma = \{\phi, \neg\psi\}$ is unsatisfiable. (4*p*)

Assignment 2 (10*p*) Prove the following claims using semantic tableaux:

(a) $\models (B \rightarrow \neg A) \wedge (B \vee C) \wedge (C \rightarrow A) \rightarrow (A \leftrightarrow C)$

(b) $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \not\models \forall y Q(y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10*p*) Derive a Prenex normal form and a clausal form (i.e. a set of clauses *S*) for the sentence

$$\neg \exists x (\exists y \neg R(x,y) \rightarrow \exists z \neg R(z,x)).$$

Make *S* as simple as possible. Prove that *S* is unsatisfiable using resolution.

Assignment 4 (10*p*) Let us represent strings “”, “*a*”, “*b*”, “*aa*”, “*ab*”, “ba”, “bb”, ... that consist of letters *a* ja *b* using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol *e*, which represents the empty string “”, and unary functions $a(x)$ and $b(x)$, that append the respective letter *a* or *b* at the beginning of a string *x*. Thus $a(b(e))$ is interpreted as $a(b(\text{“”})) = a(\text{“}b\text{”}) = \text{“}ab\text{”}$.

(a) Define predicate $O(x) =$ “the number of occurences of *a* in the string *x* is odd” using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models O(a(b(a(e)))).$$

Assignment 5 (10*p*)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

$$\texttt{v=x;z=y;while(!(z==0))\{z=z-1;v=v-1\}}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.