Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

**Assignment 1** (10p)

(a) Define the following concepts: **theorem**, **contradictory path**, and **structure**. (3 × 2p)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that if $\phi \equiv \psi$, then $\phi \land \chi \equiv \psi \land \chi$ for any sentence $\chi$. (4p)

**Assignment 2** (10p) Prove the following claims using semantic tableaux:

(a) $\not\models (A \rightarrow B) \leftrightarrow (A \rightarrow (B \rightarrow C))$

(b) $\{\forall x (P(x) \rightarrow R(x)), \forall x (\neg Q(x) \rightarrow \neg R(x))\} \models \forall x (P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses $S$) for the sentence

$\neg (\exists x (A(x) \lor B(x)) \rightarrow \exists x A(x) \lor \exists x B(x))$.

Make $S$ as simple as possible. Prove that $S$ is unsatisfiable using resolution.

**Assignment 4** (10p) Let us consider a stack of books which is described using a binary predicate $T(x,y) =$ “book $x$ is immediately on top of book $y$ in the stack”.

Suppose that $b$, $c$, and $d$ are three constants referring to specific books authored by Böll, Carr, and Dostojevski, respectively.

(a) Define a ternary predicate $B(x,y,z) = \text{"book } y \text{ appears between books } x \text{ and } z, \text{out of which } x \text{ appears higher in the stack than } y \text{ and } z"$ using predicate logic so that your definition covers all books in an individual stack.

(b) Give a model $S \models \Sigma$ of your definition $\Sigma$ on the basis of which it holds that

$\Sigma \cup \{T(b,c), T(c,d)\} \not\models B(d,c,b)$.

**Assignment 5** (10p)

Explain how the **weakest precondition** $B_1$ of an if-statement

if($B$) then {$C_1$} else {$C_2$}

can be formed given a postcondition $B_2$ for it.

Consider the following program Minus:

$v = x$; $z = y$; while(!$z == 0$) {$z = z - 1$; $v = v - 1$}.

Use weakest preconditions and a suitable invariant to establish

$\models_p [true] \text{Minus} [v == x - y]$.