T-79.3001 Logic in Computer Science: Foundations

Examination, August 18, 2008

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *theorem*, *contradictory path*, and *structure*. (3 × 2p)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that if $\phi \equiv \psi$, then $\phi \wedge \chi \equiv \psi \wedge \chi$ for any sentence χ. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\not\models ((A \to B) \to C) \leftrightarrow (A \to (B \to C))$

(b) $\{\forall x(P(x) \to R(x)), \forall x(\neg Q(x) \to \neg R(x))\} \models \forall x(P(x) \to Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x(A(x) \vee B(x)) \to \exists x A(x) \vee \exists x B(x)).$$

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us consider a stack of books which is described using a binary predicate $T(x,y)$ = "book x is immediately on top of book y in the stack". Suppose that b, c, and d are three constants referring to specific books authored by Böll, Carr, and Dostojevski, respectively.

(a) Define a ternary predicate $B(x,y,z)$ = "book y appears between books x and z, out of which x appears higher in the stack than y and z" using predicate logic so that your definition covers all books in an individual stack.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \cup \{T(b,c), T(c,d)\} \not\models B(d,c,b).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then \{}C_1\texttt{\} else \{}C_2\texttt{\}}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=x; z=y; while(!(z==0)) {z=z-1; v=v-1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.