Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: ground term, modus ponens, and the scope of a quantifier. (3 × 2p)

(b) What is meant by the notation \( C_n(\Sigma) \)?

Prove in detail that if \( \Sigma_1 \subseteq \Sigma_2 \), then \( C_n(\Sigma_1) \subseteq C_n(\Sigma_2) \). (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) \( \models (A \rightarrow B) \land (B \rightarrow C) \land (C \rightarrow A) \rightarrow (A \leftrightarrow C) \)

(b) \( \models \forall x \exists y (P(x) \land Q(y)) \rightarrow \exists y \forall x (P(x) \land Q(y)) \)

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses \( S \)) for the sentence

\[ \neg \exists x \forall y ( \forall z R(x, z) \rightarrow \forall x R(x, y)) \]

Make \( S \) as simple as possible. Prove that \( S \) is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers 0, 1, 2, ... using ground terms 0, \( s(0) \), \( s(s(0)) \), ... built of a constant symbol 0 and a function symbol \( s \) which is interpreted as the function \( s(x) = x + 1 \) for natural numbers \( x \).

(a) Define a predicate \( D(x, y, z) = \) “the distance between numbers \( x \) and \( y \) is \( z \)” using sentences of predicate logic so that your definition covers all natural numbers (represented in the way explained above).

(b) Give a model \( S \models \Sigma \) of your definition \( \Sigma \) on the basis of which it holds that

\[ \Sigma \not\models \exists x \exists y (D(x, x, x) \land D(y, y, y) \land \neg (x = y)) \]

Assignment 5 (10p)

Explain how the weakest precondition \( B_1 \) of an if-statement

\[
\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}
\]

can be formed given a postcondition \( B_2 \) for it.

Consider the following program Minus:

\[
v = x; z = y; \text{while}(!(z == 0)) \{z = z - 1; v = v - 1\}.
\]

Use weakest preconditions and a suitable invariant to establish

\[ \models_p [\text{true}] \text{ Minus } [v == x - y]. \]

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.