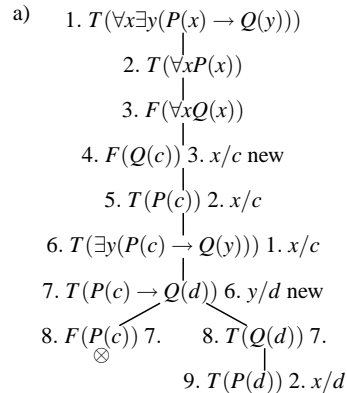


Solutions to demonstration problems

4. Use semantic tableaux to see whether the following claims holds.

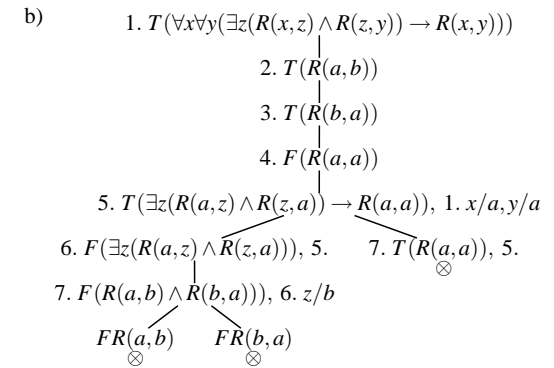
- a)  $\{\forall x\exists y(P(x) \rightarrow Q(y)), \forall xP(x)\} \models \forall xQ(x)$   
 b)  $\{\forall x\forall y(\exists z(R(x,z) \wedge R(z,y)) \rightarrow R(x,y)), R(a,b), R(b,a)\} \models R(a,a)$

Solution.



It seems that the tableaux cannot be finished. We read a counter-example  $\mathcal{S}$  from an open branch: domain  $U = \{1, 2\}$ , interpretations for constants  $c^{\mathcal{S}} = 1$  and  $d^{\mathcal{S}} = 2$ , and interpretations for predicates  $P^{\mathcal{S}} = \{1, 2\}$  and  $Q^{\mathcal{S}} = \{2\}$ .

Since the tableau is not finished, we need to check the counter-example. Now, we get  $\mathcal{S} \models \forall x\exists y(P(x) \rightarrow Q(y))$ ,  $\mathcal{S} \models \forall xP(x)$  and  $\mathcal{S} \not\models \forall xQ(x)$  for  $\mathcal{S}$ .



All the branches in the tableau are contradictory and thus the claim holds.

5. We know that

- (i) All guilty persons are liars.
- (ii) At least one of the accused is also a witness.
- (iii) No witness lies.

Use semantic tableaux to prove that all accused are not guilty.

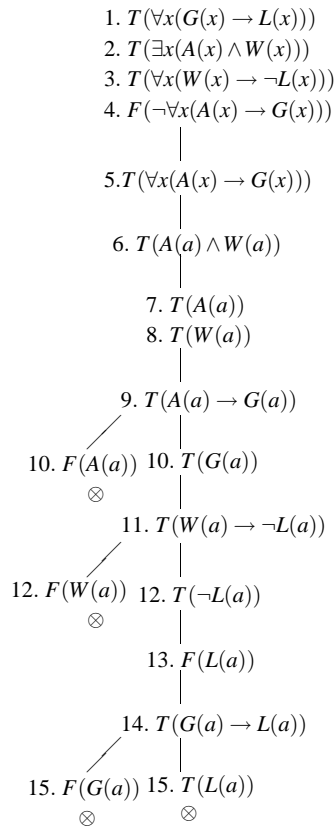
Solution. We choose the following predicates:

- $G(x)$  = “ $x$  is guilty”,
- $L(x)$  = “ $x$  is liar”,
- $A(x)$  = “ $x$  is accused”, and
- $W(x)$  = “ $x$  is witness”.

The sentences are:

- (i)  $\forall x(G(x) \rightarrow L(x))$ ,
- (ii)  $\exists x(A(x) \wedge W(x))$ , ja
- (iii)  $\forall x(W(x) \rightarrow \neg L(x))$ .

and we want to show that  $\neg\forall x(A(x) \rightarrow G(x))$ . The tableaux proof is as follows.



6. We know that:

- 1) If a brick is on another brick, then it is not on the table.
- 2) Every brick is either on the table or on another brick.
- 3) No brick is on a brick which is also on some other brick.

Use semantic tableaux to prove that if a brick is on another brick, the other brick is on the table.

**Solution.** We use the following predicates:

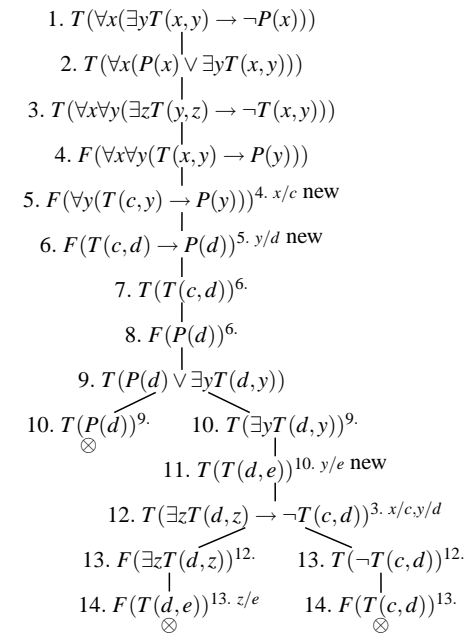
$T(x, y)$  = “brick  $x$  is on brick  $y$ ”, and  
 $P(x)$  = “brick  $x$  is on the table”.

The set of sentences is:

$$\{\forall x(\exists y T(x, y) \rightarrow \neg P(x)), \forall x(P(x) \vee \exists y T(x, y)), \forall x \forall y(\exists z T(y, z) \rightarrow \neg T(x, y))\}$$

and we want to show that  $\forall x \forall y(T(x, y) \rightarrow P(y))$ .

Tableau proof:



Note: 1) can be equivalently stated as  $\forall x \forall y(T(x, y) \rightarrow \neg P(x))$  and 3) as  $\forall x \forall y \forall z(T(y, z) \rightarrow \neg T(x, y))$ . How would the tableau look if you used these sentences?