T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 9 ([NS, 1997], Predicate Logic, Chapters 6 – 7)
April 3–4, and 12, 2007

Tutorial problems

1. Proof the following claims using semantic tableaux.

a) $\models \forall x(P(x) \leftrightarrow \neg Q(x)) \leftrightarrow \neg\exists x(P(x) \leftrightarrow Q(x))$

b) $\{\forall x\forall y(R(x,y) \rightarrow R(y,x))\} \not\models \forall x R(a,x)$

c) $\{\forall x\neg(A(x) \leftrightarrow B(x)), \forall y A(y) \vee \forall y\neg A(y)\} \models \forall z B(z) \vee \forall z\neg B(z)$

2. Use semantic tableaux to see whether the following claims holds.

a) $\models \forall x\exists y R(x,y) \rightarrow (\forall y(\neg S(y) \rightarrow \neg\exists x R(x,y)) \rightarrow \exists x S(x))$

b) $\{\exists x\exists y P(x,y), \forall x\forall y(P(x,y) \rightarrow Q(x,y))\} \models \exists x Q(x,x)$

3. We define predicates $L(x,y)$ (there is a flight from city x to city y) and $Y(x,y)$ (there is a connection from city x to city y) in following way:

$$\forall x\forall y(L(x,y) \rightarrow L(y,x))$$
$$\forall x\forall y(L(x,y) \rightarrow Y(x,y))$$
$$\forall x\forall y\forall z(Y(x,y) \wedge L(y,z) \rightarrow Y(x,z))$$

Write a database which states that there is a flights from Helsinki to London, Rovaniemi and New York, that there is a flight from New York to Paris and that there is a flight from Petersburg to Berlin. Use semantic tableaux to prove that there is a connection between Rovaniemi and New York.

Demonstration problems

4. Use semantic tableaux to see whether the following claims holds.

a) $\{\forall x\exists y(P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall x Q(x)$

b) $\{\forall x\forall y(\exists z(R(x,z) \wedge R(z,y)) \rightarrow R(x,y)), R(a,b), R(b,a)\} \models R(a,a)$

c) $\models \forall x\exists y R(x,y) \rightarrow (\forall y(\neg S(y) \rightarrow \neg\exists x R(x,y)) \rightarrow \exists x S(x))$

5. We know that

(i) All guilty persons are liars.

(ii) At least one of the accused is also a witness.

(iii) No witness lies.

Use semantic tableaux to prove that all accused are not guilty.

6. We know that:

1) If a brick is on another brick, then it is not on the table.
2) Every brick is either on the table or on another brick.
3) No brick is on a brick which is also on some other brick.

Use semantic tableaux to prove that if a brick is on another brick, the other brick is on the table.