T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 6 ([Nerode and Shore, 1997], Chapter 8)
March 13–15, 2007

Tutorial problems

1. Give a resolution proof for the following claims.

a) $\models (\neg R \vee (P \wedge Q)) \rightarrow (R \rightarrow P) \wedge (R \rightarrow Q)$

b) $A \leftrightarrow B$ is logically equivalent to $(A \wedge B) \vee (\neg A \wedge \neg B)$.

2. a) Give a resolution proof for

$$\models \neg((A \leftrightarrow (B \rightarrow C)) \wedge ((A \leftrightarrow B) \wedge (A \leftrightarrow \neg C))).$$

b) Use semantic tableux to prove

$$\models \neg((A \leftrightarrow (B \rightarrow C)) \wedge ((A \leftrightarrow B) \wedge (A \leftrightarrow \neg C))).$$

3. Recall Exercise 3 in Tutorial 3 where you were asked to formalize the following text using propositional logic:

"If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned."

Give a resolution proof for the proposition "a unicorn is magical".

Demonstration problems

4. A few weeks ago a traffic light system was modeled. Transform the propositions specifying the behaviour of the system into clausuls and prove with resolution that both red lights are not on at the same time.

5. One successful application of expert systems has been analyzing the problem of which chemical syntheses are possible. Consider the following chemical reactions:

(1) $MgO + H_2 \rightarrow Mg + H_2O$

(2) $C + O_2 \rightarrow CO_2$

(3) $CO_2 + H_2O \rightarrow H_2CO_3$

a) Represent these rules and the assumptions that we have some MgO, H_2, O_2 and C by propositional logic formulas.

b) Give a resolution proof that we can get some H_2CO_3.

6. Construct a deterministic Turing machine that counts the successor of a given binary number.

7. Show the problem of 3-coloring a graph is in the class **NP** by reducing it into the propositional satisfiability problem.