T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 4 ([Nerode and Shore, 1997], Chapters 4 and 7)
February 13–15, 2007

The first periodic time tracking questionnaire is open 9th–16th February at `http://www.cs.hut.fi/cgi-bin/teekysely.pl?action=showform&id=T793001-T-79.3001_2007ajankaytto1`

If you answer all the questionnaires in time, you get two bonus points for the exam, see `http://www.tcs.hut.fi/Studies/T-79.3001/2007SPR/index.shtml#feedback` for more details.

Tutorial problems

1. Use semantic tableaux to prove the following:

a) $\models \neg C \to (\neg A \vee (B \to \neg(C \leftrightarrow B)))$

b) $\{P \wedge (Q \vee R)\} \models (P \wedge Q) \vee (P \wedge R)$

c) $\{Q \to \neg P, P \to R\} \models (Q \to P) \to \neg Q$

2. Use a semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a) $\models (P \vee Q \vee \neg R) \wedge ((\neg R \vee Q \vee P) \to (R \vee Q) \wedge \neg Q \wedge \neg P)$

b) $\{A \to B \wedge C, \neg B\} \models \neg A$

c) $\{\neg A \wedge \neg B \leftrightarrow C \vee D, \neg C \wedge \neg D\} \models A \wedge B$

3. Give a Hilbert style proof for

$$\{P \to Q, \neg Q\} \vdash \neg P.$$

Demonstration problems

4. Peirce arrow is defined as:

$$A \downarrow B \Leftrightarrow_{def} \neg A \wedge \neg B.$$

Define semantic tableaux rules for it.

5. Use semantic tableux to show that the following propositions are valid.

a) $A \to (B \to B)$,

b) $(A \to B) \land (B \to C) \to (A \to C)$,

c) $(A \to B) \land (A \to C) \to (A \to B \land C)$ ja

d) $(A \to C) \land (B \to C) \land (A \lor B) \to C$.

6. Use semantic tableaux to check whether the following claims hold. If not, give a counterexample.

a) $\{B \to A, C \to B, (C \to A) \to D\} \models D$

b) $\{A \to C, A \lor B, \neg D \to \neg B\} \models C \to D$

c) $\models (A \to (B \to C)) \to ((A \to C) \to (A \to B))$

d) $\models (\neg B \to (A \to C)) \to (A \to (B \lor C))$

7. Recall the specification for two traffic light posts positioned in the intersection of two one-way streets discussed earlier in tutorials. Use semantic tableaux to prove that "the red lights can't be on at the same" is a logical consequence of the set of propositions describing the behaviour of the system.

8. Use the proof system by Hilbert to prove the following.

a) $\vdash P \to P$

b) $\{P \to Q, Q \to R\} \vdash P \to R$

c) $\{P, Q \to (P \to R)\} \vdash Q \to R$