T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 10 ([NS, 1997], Predicate Logic, Chapters 6 – 7)
April 17–19, 2007

The third periodic time tracking questionnaire is open 13th–20th April at `http://www.cs.hut.fi/cgi-bin/teekysely.pl?action=showform&id=T793001-T-79.3001_2007ajankaytto3`

If you answer all the questionnaires in time, you get two bonus points for the exam, see `http://www.tcs.hut.fi/Studies/T-79.3001/2007SPR/index.shtml#feedback` for more details.

Tutorial problems

1. Use semantic tableaux to prove that the last sentence is not a logical consequence of the first two sentences.

> "Penguins are black and white. Some old tv shows are black and white. Therefore, some penguins are old tv shows."

2. Let R be a binary predicate with interpretation $R^{\mathcal{S}} \subseteq U \times U$ (the set U is the domain of structure $\mathcal{S}$). In the following table we give definitions for some properties of relation $R^{\mathcal{S}}$.

Property	Definition
reflexivity	$\forall x R(x,x)$
irreflexivity	$\forall x \neg R(x,x)$
symmetry	$\forall x \forall y (R(x,y) \rightarrow R(y,x))$
asymmetry	$\forall x \forall y (R(x,y) \rightarrow \neg R(y,x))$
transitivity	$\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \rightarrow R(x,z))$
seriality	$\forall x \exists y R(x,y)$

Use semantic tableaux to show that an irreflexive and transitive relation is also asymmetric.

3. Formalize the following sentences using predicate logic and use semantic tableaux to prove that sentence 4 is a logical consequence of sentences 1-3.

1. Alders are broad-leaved trees.
2. Trees are alders, spruces or pines.
3. Spruces and pines are coniferous trees.

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4. Trees are coniferous or broad-leaved.

Demonstration problems

4. A *directed* graph consists of a set of nodels and a set of *directed* egdes between the nodes. Assume that nodes a represented with constants $\{a,b,\ldots\}$ and edges with a binary predicate $K(x,y)$ = “there is an edge from node x to node y“.

1. Define predicates $R_n(x,y) =$ “node y is reachable from node x using n edges”, for $n = 0,1,2,\ldots,k$. Represent the following graph with predicate K.

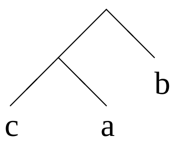
$$a \;\substack{\longrightarrow \\ \longleftarrow}\; b \longrightarrow c$$

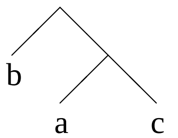
2. Use semantic tableaux to show that

$$\exists x(R_2(x,x) \wedge R_3(x,c))$$

is a logical consequence of the representation of the graph and definitions of predicates R_2 and R_3

5. We represent binary trees using a binary function s (internal nodes) and a unary function l (leaf nodes). Thus the representation of the upper tree is the term $s(s(l(c),l(a)),l(b))$.





a) Let predicate PK(x,y) denote that binary tree x is the mirror-image of binary tree y. Define predicate PK.

b) Use semantic tableaux to proof that the upper binary tree is the mirror-image of the lower binary tree.

6. Quantifier $\exists! x$ is used to denote “there is only one x”. Sentence $\exists! x\,\phi(x)$ can be represented as

$$(\exists x \phi(x)) \wedge (\forall x \forall y(\phi(x) \wedge \phi(y) \rightarrow x = y)).$$

Formalize the following sentences using predicate logic:

1. There is only one Father Christmas.
2. Every Santa Claus is Father Christmas.
3. Every Father Christmas is Santa Claus.

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4. There is only one Santa Clause.

Use semantic tableaux to prove that sentence 4 is a logical consequence of sentences 1-3.