T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 1 (preliminaries 3.1 – 3.2, propositional logic 1.1 – 1.5)
January 23–25, 2007

Tutorial problems

1. Prove by induction that $4^n - 1$ is divisible by three for all $n \geq 0$. (Hint: a number is divisible by three if it's of the form $3m$ for some integer m.)

2. Formalize the following statements in propositional logic:

a) I'll have coffee or tea and a sandwich.

b) If it rains or the wind is too heavy, we won't go out.

c) Either John or Mary will pick up Lisa from day care in the evening.

d) There's no smoke without fire.

e) When it isn't raining I walk to work, otherwise I drive my car.

3. Remove unnecessary parenthesis from the following propositional statements. What are the forms of the statements? Give parse trees for the propositions.

a) $(((A \wedge (\neg B)) \leftrightarrow C) \vee (A \rightarrow (\neg C)))$

b) $(\neg(((\neg B) \vee (\neg D)) \rightarrow (B \vee (C \vee (\neg A)))))$

c) $(A \leftrightarrow (D \vee ((B \rightarrow (\neg D)) \wedge C)))$

Demonstration problems

4. Prove by induction that a set of n elements has 2^n subsets.

5. Prove the following claims (sets A, B and C are subsets of universe E):

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

b) $E - (A \cap B) = (E - A) \cup (E - B)$.

6. Formalize the following statements in propositional logic:

a) I can't finish my work unless you help me.

b) I either walk, ride a bicycle, or sometimes drive a car to work.

c) Merja and Arto are coming to visit us.

d) You won't get dessert because you have been naughty

e) Even though the manual was long I finished reading it too early.

f) If somebody asks me — or even if no one does — he shouldn't buy a car or he must live far from his workplace and gasoline should become cheaper.

7. Let $\mathcal{P} = \{A, B, C\}$ be the set of atomic propositions. Which of the following are propositional statements? Why?

a) A

b) $\neg(A \wedge B))$

c) $(A \wedge (B \rightarrow (A \wedge C)))$

d) It is raining today.

8. Prove that all propositional statements have an even number of parenthesis.

9. Remove unnecessary parenthesis so that the meaning of the proposition does not change.

a) $(A \rightarrow ((B \wedge C) \vee D))$

b) $(((A \rightarrow B) \wedge (B \rightarrow C)) \rightarrow (A \rightarrow C))$

c) $((A \wedge (B \vee C)) \vee (A \wedge (C \vee D)))$

d) $((\neg(A \wedge B)) \leftrightarrow ((B \rightarrow C) \wedge A))$

e) $(((\neg A) \wedge (\neg B)) \rightarrow \neg(A \vee B))$

10. What are the forms of the propositional statements in the previous exercise? Give parse trees for the propositions.

11. List the substatements of the following propositional statement.

$$(\neg A \rightarrow (\neg B \rightarrow C)) \rightarrow (\neg(\neg A \rightarrow B) \rightarrow C)$$