T-79.3001 Logic in Computer Science: Foundations
Examination, March 6, 2008

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *axiom*, *structure*, and *unique names assumption*. (3 × 2p)

(b) What is meant by the notation $Cn(\Sigma)$?

Prove in detail that if $\Sigma_1 \subseteq \Sigma_2$, then $Cn(\Sigma_1) \subseteq Cn(\Sigma_2)$. (4$p$)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow B) \wedge (B \rightarrow C) \wedge (C \rightarrow A) \rightarrow (A \rightarrow C) \wedge (C \rightarrow B) \wedge (B \rightarrow A)$

(b) $\models \exists x(P(x) \vee Q(x)) \rightarrow \exists x P(x) \vee \exists x Q(x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg \exists x \forall y (\forall z R(x,z) \rightarrow \forall x R(x,y)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms $0, s(0), s(s(0)), \ldots$ built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x+1$ for natural numbers x.

(a) Define predicates $D(x) = $ "x is is divisible by 3" ja $I(x) = $ "x is indivisible by 3" using predicate logic so that your definition covers all natural numbers represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models \exists x(D(x) \wedge I(x)).$$

Assignment 5 (10p)

Explain how the *weakest precondition B_1* of an if-statement

$$\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Divide`:

$$v = 0; z = x; \text{while}(z >= y) \{z = z - y; v = v + 1\}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}] \text{ Divide } [\texttt{v == x / y}],$$

where `x / y` denotes the integer quotient when `x` is divided by `y`.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.