Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)
(a) Define the following concepts: disjunctive normal form, complete proof system, and most general unifier. (3 × 2p)
(b) What is meant by the notation $\phi \lor \psi$?
Prove in detail that if $|\phi \lor \psi| = \phi$, then $|\phi \lor \psi| = \neg \phi \lor \neg \psi$.

Assignment 2 (10p) Prove the following claims using semantic tableaux:
(a) $|P \rightarrow Q \lor R| \iff |\neg Q \rightarrow \neg P \land \neg R|
(b) \{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \models \exists z Q(z)
Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses $S$) for the sentence
\[\neg \forall x \exists y (\exists z R(y,z) \rightarrow \exists v R(x,v))\]
Try to make $S$ as simple as possible. Prove that $S$ is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings “”, “a”, “b”, “aa”, “ab”, “ba”, “bb”, … that consist of letters a ja b using ground terms
\[e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), \ldots,\]
built of a constant symbol $e$, which represents the empty string “”, and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string $x$. Thus $a(b(e))$ is interpreted as $a(b(\text{“”})) = a(\text{“b”}) = \text{“ab”}$. 
(a) Define predicate $AB(x) = \text{“the string } x \text{ is of the form abab…ab where the}\\
\begin{cases} \text{string } ab \text{ repeats } n \geq 0 \text{ times} \end{cases}$ using predicate logic so that your definition covers all finite strings represented as explained above.
(b) Give a model $S |\models \Sigma$ of your definition $\Sigma$ on the basis of which it holds that
\[\Sigma \not|\models AB(b(a(e))).\]

Assignment 5 (10p)
Explain how the weakest precondition $B_1$ of an if-statement
\[\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}\]
can be formed given a postcondition $B_2$ for it.
Consider the following program Divide:
\[v=0; z=x; \text{while}(z>y) \{z=z-y; v=v+1\}.\]
Use weakest preconditions and a suitable invariant to establish
\[|\models_p \text{true} \text{ Divide }[v=x/y],\]
where $x/y$ denotes the integer quotient when $x$ is divided by $y$. 

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.