Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10 p)
(a) Define the following concepts: formation tree, free variable occurrence, and unique names assumption. (3 × 2 p)
(b) What is meant by the notation $\phi \equiv \psi$?
Prove in detail that if $|\models \phi \rightarrow \psi$ and $|\models \neg \phi \rightarrow \neg \psi$, then $\phi \equiv \psi$.

Assignment 2 (10 p) Prove the following claims using semantic tableaux:
(a) $|\not\models (\neg A \leftrightarrow B \lor C) \leftrightarrow (A \leftrightarrow \neg B \lor \neg C)$
(b) $|\models \exists x \forall y Q(x, y) \rightarrow \forall y \exists x Q(x, y)$
Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10 p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses $S$) for the sentence
$$\neg \forall x \exists y (\exists z R(x, z) \rightarrow \exists y R(y, v)).$$
Try to make $S$ as simple as possible. Prove that $S$ is unsatisfiable using resolution.

Assignment 4 (10 p) Let us represent any finite string consisting of letters $a$, $b$, and $c$ using unary function symbols $a$, $b$, and $c$ and a constant symbol $e$ denoting the empty string. Thus, for instance, the term $a(x)$ denotes a string that starts with an $a$ followed by a string $x$, and the ground term $b(a(b(a(e))))$ represents “baba”.
(a) Define the predicate $L(x, y) =$ “string $x$ strictly precedes string $y$ in the lexicographic order” so that your definition covers all finite strings represented as described above.
(b) Give a model $S |\models \Sigma$ for your definition $\Sigma$ according to which it holds that
$$\Sigma |\not\models \exists x \exists y (L(x, y) \land L(y, x)).$$

Assignment 5 (10 p)
Explain how the weakest precondition $B_1$ of an if-statement
$$\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}$$
can be formed given a postcondition $B_2$ for it.
Consider the following program Divide:
$$v=0; z=x; \text{while}(z>y) \{z=z-y; v=v+1\}.$$  
Use weakest preconditions and a suitable invariant to establish
$$|\models_p \text{true} \ \text{Divide} [v=x/y],$$
where $x/y$ denotes the integer quotient when $x$ is divided by $y$.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.