T-79.3001 Logic in Computer Science: Foundations

Examination, August 13, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 $(10p)$

(a) Define the following concepts: *theorem*, *Herbrand universe*, and *composition of substitutions*. $(3 \times 2p)$

(b) What is meant by the notation $\models \phi$?

Prove in detail that if $\models \phi$ and $\models \phi \rightarrow \psi$, then $\models \psi$.

Assignment 2 $(10p)$ Prove the following claims using semantic tableaux:

(a) $\models (((A \rightarrow B) \rightarrow C) \rightarrow D) \rightarrow (A \rightarrow B \vee D) \wedge (C \rightarrow D)$

(b) $\models \exists x(P(x) \rightarrow \forall x P(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 $(10p)$ Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\forall x P(x) \vee \forall y P(y) \rightarrow \forall x \exists y (P(x) \wedge P(y))).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 $(10p)$ Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Define predicates $D(x)$ = "x is is divisble by 3" ja $I(x)$ = "x is indivisible by 3" using predicate logic so that your definition covers all natural numbers represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models \exists x(D(x) \wedge I(x)).$$

Assignment 5 $(10p)$

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\,\texttt{then}\,\{C_1\}\,\texttt{else}\,\{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Divide`:

```
v=0;z=x;while(z>=y){z=z-y;v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Divide}\ [\texttt{v==x/y}],$$

where `x/y` denotes the integer quotient when `x` is divided by `y`.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.