Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: refutation by resolution, free variable occurrence, and disagreement set. (3 × 2p)

(b) What is meant by the notation \( \Sigma \models \phi \)?

Prove in detail that if \( \Sigma \cup \{\phi\} \models \psi \), then \( \Sigma \models \phi \rightarrow \psi \).

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) \( \models (A \land B) \lor (\neg A \land C) \rightarrow \neg(A \land \neg B) \land (\neg C \rightarrow A) \)

(b) \( \models \exists x(P(x) \lor Q(x)) \leftrightarrow \exists xP(x) \lor \exists xQ(x) \)

Tableau proofs must contain all intermediary steps !!!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses \( S \)) for the sentence

\[ \neg(\exists x\forall yG(y,x) \rightarrow \forall y\exists xG(y,x)). \]

Try to make \( S \) as simple as possible. Prove that \( S \) is unsatisfiable using resolution.

Assignment 4 (10p) Let us consider the contents of a book shelf which is described using a binary predicate \( C(x,y) = \text{"books } x \text{ and } y \text{ are located consecutively on the shelf"} \). Suppose that \( b, c \) and \( d \) are three constants referring to specific books authored by Böll, Carr, and Dostojevski, respectively.

(a) Define a ternary predicate \( B(x,y,z) = \text{"book } y \text{ appears between books } x \text{ and } z, \text{ i.e., after } x \text{ but before } z \text{ on the shelf"} \) using predicate logic so that your definition covers all books on an individual shelf.

(b) Give a model \( \models \Sigma \) of your definition \( \Sigma \) on the basis of which it holds that

\[ \Sigma \cup \{C(b,c),C(c,d)\} \not\models B(d,c,b). \]

Assignment 5 (10p)

Explain how the weakest precondition \( B_1 \) of an if-statement

\[
\text{if}(B) \text{ then } \{C_1\} \text{ else } \{C_2\}
\]

can be formed given a postcondition \( B_2 \) for it.

Consider the following program Divide:

\[
v=0; z=x; \text{while}(z>y) \{ z=z-y; v=v+1 \}.\]

Use weakest preconditions and a suitable invariant to establish

\[ \models_p \text{true} \text{ Divide } [v=x/y], \]

where \( x/y \) denotes the integer quotient when \( x \) is divided by \( y \).

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.


Please remember the last time tracking questionnaire!