T-79.3001 Logic in Computer Science: Foundations

Examination, March 5, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 ($10p$)

(a) Define the following concepts: *tautology*, *disjunctive normal form*, and *Herbrand base*. ($3 \times 2p$)

(b) What is meant by the notation $\mathcal{A} \models \phi$?

Prove in detail that for every $\mathcal{A}$ and ϕ, $\mathcal{A} \models \phi$ or $\mathcal{A} \not\models \phi$. ($4p$)

Assignment 2 ($10p$) Prove the following claims using semantic tableaux:

(a) $\models \neg(A \wedge \neg B) \wedge (\neg C \rightarrow A) \rightarrow (A \wedge B) \vee (\neg A \wedge C)$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(Q(x) \rightarrow R(x))\} \not\models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 ($10p$) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x A(x) \vee \exists x B(x) \rightarrow \exists x(A(x) \vee B(x))).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 ($10p$) Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Define predicates $D(x) =$ "x is is divisble by 3" ja $I(x) =$ "x is indivisible by 3" using predicate logic so that your definition covers all natural numbers represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models \exists x(D(x) \wedge I(x)).$$

Assignment 5 ($10p$)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\;\texttt{then}\;\{C_1\}\;\texttt{else}\;\{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=0;v=v-x;z=y;while(!(z==0)){z=z-1;v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\;\texttt{Minus}\;[\texttt{v==y-x}].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.