Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, January 3, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *truth assignment*, *sound proof system*, and *ground term*. ($3 \times 2p$)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that $\phi \equiv \psi$ if and only if $\models \phi \leftrightarrow \psi$. (4$p$)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \wedge B) \vee (\neg A \wedge C) \rightarrow (A \rightarrow B) \wedge (\neg A \rightarrow C)$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\forall x A(x) \wedge \forall x B(x) \rightarrow \forall x(A(x) \wedge B(x))).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings "", "a", "b", "aa", "ab", "ba", "bb", ... that consist of letters a ja b using ground terms

$$e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), \ldots,$$

built of a constant symbol e, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(\text{“”})) = a(\text{“}b\text{”}) = \text{“}ab\text{”}$.

(a) Define predicate $O(x) =$ "the number of occurences of a in the string x is odd" using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models O(a(b(a(e)))).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then \{}C_1\texttt{\} else \{}C_2\texttt{\}}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Diff`:

```
v=0; z=0; while(!(z==y)) {z=z+1; v=v-1}; v=v+x.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Diff}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.