Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, November 1, 2006

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *formation tree*, *atomic formula*, and *renaming substitution*. ($3 \times 2p$)

(b) What is meant by the notation $\models \phi$?

Prove in detail that if $\models \phi$ and $\models \neg\phi \vee \psi$, then $\models \psi$.

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \to B) \wedge (\neg A \to C) \to (A \wedge B) \vee (\neg A \wedge C)$

(b) $\{\forall x \exists y(P(x) \to Q(y)), \forall x P(x)\} \not\models \forall y Q(y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(A(c) \wedge \forall x(\neg B(x) \to \neg A(x)) \to \exists x B(x)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent strings "", "a", "b", "aa", "ab", "ba", "bb", ... that consist of letters a ja b using ground terms

$$e, a(e), b(e), a(a(e)), a(b(e)), b(a(e)), b(b(e)), \ldots,$$

built of a constant symbol e, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(\text{“”})) = a(\text{“}b\text{”}) = \text{“}ab\text{”}$.

(a) Define predicate $AB(x) =$ "the string x is of the form $abab\ldots ab$ where the string ab repeats $n \geq 0$ times" using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models AB(b(a(e))).$$

Assignment 5 (10p) Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then \{}C_1\texttt{\} else \{}C_2\texttt{\}}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Subtract`:

```
z=0; v=x; while(!(z==y)) {z=z+1; v=v-1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Subtract}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.