Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, May 16, 2006

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *disjunctive normal form*, *complete proof system*, and *most general unifier*. ($3 \times 2p$)

(b) What is meant by the notation $\Sigma \models \phi$?

Prove in detail that if $\Sigma \models \phi$ and $\Sigma \subseteq \Gamma$, then $\Gamma \models \phi$.

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (((A \to B) \to C) \to D) \to (A \to B \vee D) \wedge (C \to D)$

(b) $\models \forall x(P(x) \to Q(x)) \to \neg\exists y(\neg Q(y) \wedge P(y))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x(A(x) \vee B(x)) \to \exists x A(x) \vee \exists x B(x)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Define predicates $O(x) =$ "x is odd" ja $E(x) =$ "x is even" using predicate logic so that your definition covers all natural numbers.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models \exists x(O(x) \wedge O(s(x))).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then \{}C_1\texttt{\} else \{}C_2\texttt{\}}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Minus`:

```
v=0; v=v-x; z=y; while(!(z==0)) {z=z-1; v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Minus}\ [\texttt{v==y-x}].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.