This is the last set of tutorial problems. The course exam is on Wednesday 4 May, 12–3 p.m., Lecture Hall T1. Please register for the exam via the TOPI course information system. The exam will be "open book", meaning that you can take a copy of your lecture notes, together with the tutorial problem sets and their solutions to the exam.

1. Consider an Ising spin system with two spins $\sigma_1, \sigma_2 \in \{-1, +1\}$ and Hamiltonian function $H(\sigma_1, \sigma_2) = -J\sigma_1\sigma_2$, $J \in \mathbb{R}$. Calculate, at given values of the coupling coefficient J and inverse temperature β, the partition function of the system, the magnetisation of each spin, and the energy of the system. (Note that the latter two quantities are expectations w.r.t. the appropriate Gibbs distribution.) Assume then that the coupling coefficient J is a random variable with density $\rho(J)$. Write down the expressions for the "quenched" averages w.r.t. J of the magnetisation and energy. What is the value of the (quenched) average ground state energy? Consider explicitly the special cases where the distibution of J is (a) uniform in an interval $[-J_0, +J_0]$, (b) normal with zero mean and unit variance.

2. [Toy "analytic continuation" example analogous to the replica method.] Consider Newton's series expansion for $(1+x)^r$, which for integer $r \geq 0$ yields the familiar binomial formula. Show that as $r \to 0$, this leads to the series expansion in x of $\ln(1+x)$.