T-79.250 **Spring 2005**
Combinatorial Models and Stochastic Algorithms
Tutorial 11, April 15
Problems

Please note that on 15 April there is no lecture (i.e. only the tutorial from 10-12 a.m. is held); the lecturer is away at a meeting.

1. It can be shown (cf. Brémaud, p. 231) that in a regular Markov chain the expected hitting times μ_{ij} (cf. problem 5 of tutorial 1) can be obtained from the fundamental matrix Z according to the formula $\mu_{ij} = (z_{jj} - z_{ij})/\pi_j$, where π_j denotes the stationary probability of state j. Repeat problem 2 of tutorial 2 using this technique, i.e. compute the expected time to reach state 0 from each of the other states in a simple random walk on the cyclically ordered state set $S = \{0, 1, 2, 3\}$.

2. Establish the validity of the Hastings MCMC design scheme (p. 108 of the lecture notes), i.e. show that with the given choices of acceptance probabilities, the resulting Markov chains are guaranteed to be reversible.

3. Construct a Barker-Hastings sampler for the setting of problem 2 of tutorial 3, with uniform generation probability $q = 1/n$ for each of the Hamming neighbours of a given state $\sigma \in \{0, 1\}^n$. Compare this to the Gibbs sampler designed earlier.

4. Verify the claim at the bottom of p. 107 of the lecture notes, i.e. that in the case of a reversible chain, the asymptotic variance of an MCMC estimate satisfies:

$$\lim_{n\to\infty} \frac{1}{n} \mathrm{Var}_\mu\Big(\sum_{k=1}^{n} f(X_k)\Big) = \sum_{i=2}^{r} \frac{1+\lambda_i}{1-\lambda_i} |\langle f, v_i\rangle_\pi|^2,$$

where $1 = \lambda_1 > \lambda_2 \geq \cdots \geq \lambda_r > -1$ are the eigenvalues of the transition probability matrix P, and u_i and v_i are the left and right eigenvectors associated to eigenvalue λ_i, normalised so that $u_i^T v_i = 1$.