T-79.250 **Spring 2005**

Combinatorial Models and Stochastic Algorithms

Tutorial 9, April 1

Problems

1. Compute the expected value of the clustering coefficient $\mathcal{C}(G)$ for an ER random graph $G \in \mathcal{G}(n,p)$. Give also some estimates for the expected value of the characteristic path length $\mathcal{L}(G)$.

2. Compute the clustering coefficient $\mathcal{C}(G)$, characteristic path length $\mathcal{L}(G)$, and distribution of node degrees for a circulant graph C_{nk}. (It suffices to compute these quantities asymptotically for fixed k and large n.) What is the edge density $p = e(C_{nk})/\binom{n}{2}$ for such a graph? What is the effect on $\mathcal{L}(G)$ of a single randomly added shortcut edge?

3. Compute the clustering coefficient $\mathcal{C}(G)$, characteristic path length $\mathcal{L}(G)$, and distribution of node degrees for a "caveman graph" consisting of k "caves" of r nodes each. What is the edge density p for such a graph? (Recall that a "caveman graph" is a cyclic arrangement of k appropriately modified r-cliques. It suffices to compute these quantities asymptotically for (a) fixed r and large k and (b) the case $r \sim k \sim \sqrt{n}$ for large n.)

4. Verify by direct calculation that in the simulated annealing algorithm the finite-temperature Gibbsian distributions $\pi^{(T)}$ for $T > 0$ do indeed converge pointwise to the desired limit distribution π^* as $T \to 0$.

5. Consider a simple state space graph with states $S = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$ and neighbourhood structure $N(\sigma_i) = \{\sigma_{i-1}, \sigma_{i+1}\}$, where the indices are computed modulo 4. Write down explicitly the transition probability matrix of the simulated annealing algorithm at temperature t for this system, when the function to be minimised is given by $H(\sigma_0) = 1$, $H(\sigma_1) = 2$, $H(\sigma_2) = 0$, $H(\sigma_3) = 2$. Given a cooling schedule where the temperature at step k of the algorithm is $t_k > 0$, what is the probability that the algorithm when initialised in the locally optimal state σ_0 will stay there forever? Find a sequence t_k for which this probability is nonzero. What kind of cooling schedule would, according to Theorem 6.5 (p. 93 of the notes) guarantee asymptotic convergence to the globally optimal state σ_2?

6. The NP-complete PARTITION problem is defined as follows: given a sequence of $2n$ nonnegative integers $x_1, \ldots, x_{2n}$, is there a subsequence of n numbers whose sum is exactly half the sum of the whole sequence? Formulate the task of finding approximate partitions of an integer sequence as a minimisation problem, and present a simulated annealing approach to solving it. What kinds of cooling schedules would Theorem 6.5 suggest for your algorithm in the case of input sequences consisting of $2n$ numbers from the interval $[0, N]$? (You might consider also actually implementing your algorithm and experimenting with some more realistic cooling schedules.)