T-79.250 **Spring 2005**

Combinatorial Models and Stochastic Algorithms

Tutorial 7, March 11

Problems

1. Consider the ensemble of n-bit binary strings $\Omega = \{0,1\}^n$ with Hamiltonian $H(\omega)$ = number of 1's in string ω. Compute explicitly the partition function Z_β for this ensemble, and derive expressions for its macroscopic total energy U_β and entropy S_β. Solve the equation $U_\beta = h$ for β.

2. Compute the partition function for the binary NK model where $K = 0$, and the fitness function for allele $a_i \in \{0,1\}$ is uniformly $f^i(a_i) = ca_i + b$, for given constants $c, b \in \mathbb{R}$.

3. Compute the expected number of (a) edges, (b) r-cliques (complete subgraphs K_r) in a random graph $G \in \mathcal{G}(n,p)$.

4. Derive Theorem 4.1 of the lecture notes (given any fixed graph H, a.e. $G \in \mathcal{G}(n,p)$ for $0 < p < 1$ contains an induced copy of H) from Lemma 4.2 of the notes (for any fixed $k, l \in \mathbb{N}$, a.e. $G \in \mathcal{G}(n,p)$ for $0 < p < 1$ has property Q_{kl}).

5. Prove that the graph property "G has maximum degree at least k" has a threshold function for $k \geq 1$, and compute it.

6. Prove that the graph property "G contains a d-dimensional cube" has a threshold function for $d \geq 1$, and compute it. (The "d-dimensional cube" has 2^d vertices represented as $\{0,1\}^d$, and two vertices are connected by an edge if and only if their representations differ in exactly one position.)