T-79.250 **Spring 2003**
Combinatorial Models and Stochastic Algorithms
Tutorial 2, February 7
Problems

1. Compute the partition function for the binary NK model where $K = 0$, and the fitness function for allele $a_i \in \{0, 1\}$ is uniformly $f^i(a_i) = ca_i + b$, for given constants $c, b \in \mathbb{R}$.

2. Design an algorithm for finding a globally optimal genotype in a binary NK model with $K = 1$. (The inputs to your algorithm are the "dependency graph" of the model, indicating for each locus i the one other locus $i' \neq i$ on which its fitness depends, and the associated local fitness functions $f^i(a_i, a_{i'})$, which may be viewed as 2×2 real matrices.) Estimate the efficiency of your algorithm.

3. Compute the expected number of (a) edges, (b) r-cliques (complete subgraphs K_r) in a random graph $G \in \mathcal{G}(n, p)$.

4. Derive Theorem 4.1 of the lecture notes (given any fixed graph H, a.e. $G \in \mathcal{G}(n, p)$ for $0 < p < 1$ contains an induced copy of H) from Lemma 4.2 of the notes (for any fixed $k, l \in \mathbb{N}$, a.e. $G \in \mathcal{G}(n, p)$ for $0 < p < 1$ has property Q_{kl}).

5. Prove that the graph property "G has maximum degree at least k" has a threshold function for $k \geq 1$, and compute it.

6. Prove that the graph property "G contains a d-dimensional cube" has a threshold function for $d \geq 1$, and compute it. (The "d-dimensional cube" has 2^d vertices represented as $\{0, 1\}^d$, and two vertices are connected by an edge if and only if their representations differ in exactly one position.)