T-79.250 **Spring 2003**
Combinatorial Models and Stochastic Algorithms
Tutorial 1, January 31
Problems

1. Show that in a canonical ensemble of microstates (i.e. w.r.t. the Gibbs probability distribution), one obtains
$$\langle H^2 \rangle = \frac{1}{Z} \frac{\partial^2 Z}{\partial \beta^2}$$
and, using this result, that the *specific heat* of the system,
$$C_V = \frac{\partial \langle H \rangle}{\partial T},$$
satisfies
$$C_V = \frac{\beta}{T} [\langle H^2 \rangle - \langle H \rangle^2],$$
i.e. indicates the fluctuations around the average energy value.

2. Compute the thermodynamic potential (average energy), specific heat, and entropy of a decoupled ($J = 0$) Ising system with N spins at temperature T and external field h.

3. Same as problems 1 and 2, but with respect to *magnetic susceptibility*
$$\chi = \frac{\partial \langle M \rangle}{\partial h} = \beta [\langle M^2 \rangle - \langle M \rangle^2]$$
in place of specific heat.

4. Derive an upper bound on the number of spin flips required for an N-spin SK system to converge to a metastable state under deterministic Glauber dynamics. Express this bound in terms of the size N and number M of binary patterns stored when the system is used as a Hopfield-type associative memory. (See p. 16 of the lecture notes or any reference work on neural networks for the Hebb/Hopfield pattern storage rule.)

5. Construct a 5-bit Hopfield associative memory network for the patterns $(+,+,+,+,+)$, $(+,-,-,+,-)$, and $(-,+,-,-,-)$. (See p. 16 of the lecture notes or any reference work on neural networks for the Hebb/Hopfield pattern storage rule.) Are the patterns stable states of the system's dynamics? To what state does the system converge from initial state $(+,-,+,+,+)$?

6. Estimate how many randomly generated bit patterns, where each bit is chosen to be $+1$ or -1 with equal probability and independently of all the other bits, can be stored in an N-bit Hopfield associative memory network using the standard Hebb/Hopfield storage rule, so that the patterns with high probability become stable states of the resulting system's dynamics. (You may want to consult the neural networks literature for this problem.)