A Glimpse Beyond

- $\mathbf{EXP} = \mathbf{TIME}(2^{n^k})$
- $\mathbf{NEXP} = \mathbf{NTIME}(2^{n^k})$
- If $\mathbf{P} = \mathbf{NP}$, then $\mathbf{EXP} = \mathbf{NEXP}$.
- If $f(n)$ and $g(n) \geq n$ are proper complexity functions, then $\mathbf{TIME}(f(n)) = \mathbf{NTIME}(f(n))$ implies $\mathbf{TIME}(g(f(n))) = \mathbf{NTIME}(g(f(n)))$.
- $\mathbf{EXP} = \mathbf{APSPACE}$

Succinct Problems

- A type of $\mathbf{EXP}/\mathbf{NEXP}$-complete problems: $\mathbf{EXP}$ and $\mathbf{NEXP}$ are $\mathbf{P}$ and $\mathbf{NP}$ but on *exponentially more succinct* input.
- A succinct representation of a graph with n nodes where $n = 2^b$ as a Boolean circuit C:
  C has $2b$ input gates and its value on two b bit integers i, j is **True** iff there is a edge $[i, j]$ in the graph.
- SUCCINCT HAMILTON PATH: Given a succinct representation C of a graph G_C, does G_C have a Hamilton path?
- SUCCINCT CIRCUIT SAT, SUCCINCT 3SAT and SUCCINCT HAMILTON PATH are $\mathbf{NEXP}$-complete.
- SUCCINCT CIRCUIT VALUE is $\mathbf{EXP}$-complete.

First-Order Satisfiability

- The general case undecidable
- Some decidable cases of first-order logic without function symbols:
  - Schönfinkel-Bernays class: $\exists x_1 \ldots \exists x_k \forall y_1 \ldots \forall y_l \phi$ ($\mathbf{NEXP}$-complete)
  - Ackermann class: $\exists x_1 \ldots \exists x_k \forall y \exists x_{k+1} \ldots \exists x_l \phi$ ($\mathbf{EXP}$-complete)
  - Gödel class: $\exists x_1 \ldots \exists x_k \forall y_1 \forall y_2 \exists x_{k+1} \ldots \exists x_l \phi$ ($\mathbf{NEXP}$-complete)
  - Monadic class ($\mathbf{NEXP}$-complete)

Beyond NEXP

- $\mathbf{EXPSPACE} = \mathbf{SPACE}(2^{n^k})$
- $2\text{-}\mathbf{EXP} = \mathbf{TIME}(2^{2^{n^k}})$
  $2\text{-}\mathbf{NEXP} = \mathbf{NTIME}(2^{2^{n^k}})$
- $3\text{-}\mathbf{EXP} = \mathbf{TIME}(2^{2^{2^{n^k}}})$
  ...
- $\mathbf{ELEMENTARY} = \mathbf{TIME}(2^{2^{\cdot^{\cdot^{\cdot^{2^n}}}}})$
- $\mathbf{R}$: recursive languages

- $\mathbf{NEXP}/\mathbf{EXP}$-complete problems are provably not in $\mathbf{P}$.
- The exponential hierarchy is a provably true hierarchy.
- There are natural problems even in the higher classes:
  Regular expression equivalence (with squaring) is $\mathbf{EXPSPACE}$-complete.
  Regular expression equivalence (with squaring and complement) is not even in $\mathbf{ELEMENTARY}$.

Computational complexity theory

- $\mathbf{L} \subseteq \mathbf{NL} \subseteq \mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} \subseteq \mathbf{EXP} \subseteq \mathbf{NEXP} \subseteq \mathbf{EXPSPACE} \subseteq 2\text{-}\mathbf{EXP} \subseteq \mathbf{ELEMENTARY} \subseteq \mathbf{R}$
- Reductions:
  A reduces to $B \longleftrightarrow$ "B is at least as hard as A"
  (i) Positive result: For solving A we can use an algorithm for B.
  (ii) Negative result: If A is hard then so is B.
- Complete problems:
  Hardest among those in the same class
  Least likely to belong to a lower class.

- When finding a hard problem:
  - Analyze the borderline of hardness
  - Study special cases
  - Efficient search methods and good heuristics
  - Approximations
  - Randomized algorithms
  - Local search methods