T-79.231 Parallel and Distributed Digital Systems
Homework Assignment 5, October 10, 2003 **Marko Mäkelä**

Invariant Calculus and Performance Analysis

1. Construct the incidence matrix of the place/transition system presented as the solution for the first assignment of the second tutorial. Present the places in the order boat_1, boat_2, cabbage_1, cabbage_2, goat_1, goat_2, wolf_1, wolf_2. Based on the structure of the net, one could guess that $M(x_1)+M(x_2)$ might be invariants for all $x \in \{\text{boat}, \text{cabbage}, \text{goat}, \text{wolf}\}$. Present vectors corresponding to these invariant candidates and check with matrix operations whether they are invariants.

2. You can fetch the state space presented on lecture slide 1-26 from `http://www.tcs.hut.fi/Opinnot/T-79.179/2003/k5.txt` in textual format.

   (a) Construct the transition rate matrix of the system with the rates according to Table 1. *Hint: In GNU Octave, you can initialise* `Q=zeros(18)` *and define a bunch of constants, e.g.,* `xmit=100`*. The transition* !msg0 *from state 10 to state 12 can now be written as* `Q(10,12)=xmit`*. Finally, initialize the diagonal elements:* `diag=-sum(Q,2); for x=1:18; Q(x,x)=diag(x); endfor`.

   (b) Solve the steady state distribution.

   (c) What is the average firing rate of the transition ?ack0?

   (d) How frequently will the consumer obtain a message?

3. Let the firing rate of the transition lose ack be tenfold when the acknowledgement channel contains the digit 1. Answer the previous questions for this modified system.

!msg0, !msg1	100/s
?msg0!ack, ?msg1!ack	200/s
?msg0!ack0, ?msg1!ack1	100/s
?ack0, ?ack1	1000/s
lose msg, lose ack	1/s

Table 1: The firing rates of enabled transitions.

Return the answer to the mailbox located between rooms B 336 and B 337 in the Computer Science Building, 3rd floor, by 8 p.m. on November 24, 2003. You may also return your answer in Postscript or PDF format to Jukka.Honkola@hut.fi.