Invariant Calculus and Performance Analysis

1. Construct the incidence matrix of the place/transition system presented as the solution for the first assignment of the second tutorial. Present the places in the order boat₁, boat₂, cabbage₁, cabbage₂, goat₁, goat₂, wolf₁, wolf₂. Based on the structure of the net, one could guess that \( M(x₁) + M(x₂) \) might be invariants for all \( x \in \{\text{boat}, \text{cabbage}, \text{goat}, \text{wolf}\} \). Present vectors corresponding to these invariant candidates and check with matrix operations whether they are invariants.


   (a) Construct the transition rate matrix of the system with the rates according to Table 1. Hint: In GNU Octave, you can initialise \( Q=\text{zeros}(18) \) and define a bunch of constants, e.g., \( \text{xmit}=100 \). The transition \( \text{!msg0} \) from state 10 to state 12 can now be written as \( Q(10,12)=\text{xmit} \). Finally, initialize the diagonal elements: \( \text{diag}=-\text{sum}(Q,2) \); for \( x=1:18 \);
   \( Q(x,x)=\text{diag}(x) \); endfor.

   (b) Solve the steady state distribution.

   (c) What is the average firing rate of the transition \( ?\text{ack0} \)?

   (d) How frequently will the consumer obtain a message?

3. Let the firing rate of the transition \( \text{lose ack} \) be tenfold when the acknowledgement channel contains the digit 1. Answer the previous questions for this modified system.

\[
\begin{array}{ccc}
\text{!msg0, !msg1} & 100/s \\
\text{!msg0!ack, !msg1!ack} & 200/s \\
\text{!msg0!ack0, !msg1!ack1} & 100/s \\
\text{?ack0, ?ack1} & 1000/s \\
\text{lose msg, lose ack} & 1/s \\
\end{array}
\]

Table 1: The firing rates of enabled transitions.

Return the answer to the mailbox located between rooms B 336 and B 337 in the Computer Science Building, 3rd floor, by 8 p.m. on November 24, 2003. You may also return your answer in Postscript or PDF format to Jukka.Honkola@hut.fi.