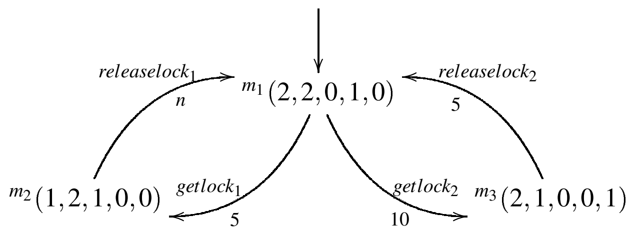
T-79.231 **Autumn 2002**
Parallel and Distributed Digital Systems
Answers to tutorial 8
Nov 11th 2002

1. First we will construct the reachability graph of the net:



Then we will create the infinitesimal generator matrix Q. (Row 1 models transition frequency from the state m_1 to the states m_2 and m_3. In the diagonal we have the negation of the sum of the rest of the elements in the row.)

$$Q = \begin{pmatrix} -15 & 5 & 10 \\ n & -n & 0 \\ 5 & 0 & -5 \end{pmatrix}$$

In the steady state the following conditions will hold: $\pi \cdot Q = 0 \wedge \sum_i \pi_i = 1$.

The steady state equations are obtained:

$$\begin{array}{rrrl} -15\pi_1 & +n\pi_2 & +5\pi_3 & = 0 \\ 5\pi_1 & -n\pi_2 & & = 0 \\ 10\pi_1 & & -5\pi_3 & = 0 \\ \pi_1 & +\pi_2 & +\pi_3 & = 1 \end{array}$$

The solution for the above system of equations is given below:

$$\pi_1 = \frac{n}{3n+5} \wedge \pi_2 = \frac{5}{3n+5} \wedge \pi_3 = \frac{2n}{3n+5}$$

With $n = 20$, we obtain the following:

$$\pi_1 = \frac{4}{13} \wedge \pi_2 = \frac{1}{13} \wedge \pi_3 = \frac{8}{13}$$

The average of tokens in the place p is obtained by addition of marking probabilities weighted with the number of tokens in the place p.

$$E[p_1] = \pi_1 \cdot m_1(p_1) + \pi_2 \cdot m_2(p_1) + \pi_3 \cdot m_3(p_1) = \frac{6n+5}{3n+5}$$

$$E[p_2] = \pi_1 \cdot m_1(p_2) + \pi_2 \cdot m_2(p_2) + \pi_3 \cdot m_3(p_2) = \frac{4n+10}{3n+5}$$

$$n = 20 \Rightarrow E[p_1] = 1\frac{12}{13} \wedge E[p_2] = 1\frac{5}{13}$$

$$n \to \infty \Rightarrow E[p_1] = 2 \wedge E[p_2] = \frac{4}{3}$$

$$n \to 0 \Rightarrow E[p_1] = 1 \wedge E[p_2] = 2$$