T-79.230 **Kevät 2005**
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Solutions

1. The CURRENT-BEST-LEARNING algorithm produces the following results. FP denotes false positive and FN denotes false negative.

   (a) $\forall x(WillWait(x) \leftrightarrow Hungry(x))$
   $x_1 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow Hungry(x)$
   $x_2 : \ FP : \ \forall x\, WillWait(x) \leftrightarrow Hungry(x) \wedge Est(x, 0-10)$
   $x_3 : \ FN : \ \forall x\, WillWait(x) \leftrightarrow Est(x, 0-10)$
   $x_4 : \ FN : \ \forall x\, WillWait(x) \leftrightarrow Est(x, 0-10) \vee Est(x, 10-30)$
   $x_5 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow Est(x, 0-10) \vee Est(x, 10-30)$
   $x_6 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow Est(x, 0-10) \vee Est(x, 10-30)$
   $x_7 : \ FP : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee Est(x, 10-30)$
   $x_8 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee Est(x, 10-30)$
   $x_9 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee Est(x, 10-30)$
   $x_{10} : \ FP : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee (Est(x, 10-30) \wedge \neg Price(x, \$\$\$))$
   $x_{11} : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee (Est(x, 10-30) \wedge \neg Price(x, \$\$\$))$
   $x_{12} : \ FN : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 0-10) \wedge Patrons(some)) \vee (Est(x, 10-30) \wedge \neg Price(x, \$\$\$)) \vee (Est(x, 30-60) \wedge Type(x, Burger))$

   (b) $\forall x(WillWait(x) \leftrightarrow \neg\, WaitEstimate(x, 30\text{–}60))$
   $x_1 : \ FN : \ \forall x\, WillWait(x) \leftrightarrow (Est(x, 30-60) \vee Pat(x, \text{some}))$
   $x_2 : \ FP : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}))$
   $x_3 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow Pat(x, \text{some})$
   $x_4 : \ FN : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee Fri(x))$
   $x_5 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee Fri(x))$
   $x_6 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee Fri(x))$
   $x_7 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee Fri(x))$
   $x_8 : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee Fri(x))$
   $x_9 : \ FP : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee (Fri(x) \wedge \neg Est(x, > 60))$
   $x_{10} : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee (Fri(x) \wedge \neg Est(x, > 60))$
   $x_{11} : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee (Fri(x) \wedge \neg Est(x, > 60))$
   $x_{12} : \ -- : \ \forall x\, WillWait(x) \leftrightarrow (Pat(x, \text{some}) \vee (Fri(x) \wedge \neg Est(x, > 60))$

2. Surprise candy comes in two flavors, cherry and lime. 4 pieces have been opened, out of which 3 were cherry.

a 100% cherry

b 75% cherry and 25% lime

c 50% cherry and 50% lime

d 25% cherry and 75% lime

e 100% lime

(a) Which is the most likely (ML) hypothesis?
Let $\mathbf{d}$ = "3 cherries and 1 lime".
$P(\mathbf{d} \mid h_a) = 0$
$P(\mathbf{d} \mid h_b) = 4 \cdot (\frac{3}{4})^3 \cdot \frac{1}{4} = \frac{27}{64} \approx 0,42$
$P(\mathbf{d} \mid h_c) = 4 \cdot (\frac{1}{2})^3 \cdot \frac{1}{2} = 0,25$
$P(\mathbf{d} \mid h_d) = 4 \cdot (\frac{1}{4})^3 \cdot \frac{3}{4} = \frac{3}{64} \approx 0,047$
$P(\mathbf{d} \mid h_e) = 0$

Thus b is the most likely hypothesis.

(b) Suppose that the prior distribution of the bags is $\langle 0.1, 0.1, 0.1, 0.6, 0.1 \rangle$.
Find out the maximum a posteriori (MAP) hypothesis
$P(h_a) = 0,1$
$P(h_b) = 0,1$
$P(h_c) = 0,1$
$P(h_d) = 0,6$
$P(h_e) = 0,1$

$P(h_a \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_a) P(h_a) = 0$
$P(h_b \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_b) P(h_b) = 0.042$
$P(h_c \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_c) P(h_c) = 0.025$
$P(h_d \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_d) P(h_d) = 0.028$
$P(h_e \mid \mathbf{d}) = \alpha P(\mathbf{d} \mid h_e) P(h_e) = 0$

$P(h_a \mid \mathbf{d}) = 0$
$P(h_b \mid \mathbf{d}) = \frac{0,042}{0,042+0,025+0,028} = 0,44$
$P(h_c \mid \mathbf{d}) = \frac{0,025}{0,042+0,025+0,028} = 0,26$
$P(h_d \mid \mathbf{d}) = \frac{0,028}{0,042+0,025+0,028} = 0,29$
$P(h_e \mid \mathbf{d}) = 0$
The maximum a posteriori hypothesis is h_b.

(c) Estimate the probability that the fifth piece of candy is lime flavored
$P(\text{lime}_5 \mid \mathbf{d}) = \frac{1}{4} P(h_b \mid \mathbf{d}) + \frac{1}{2} P(h_c \mid \mathbf{d}) + \frac{3}{4} P(h_d \mid \mathbf{d})$
Using the prior distribution of bags in part (b) gives us the probability of getting a lime from the bag in fifth take as:
$P(\text{lime}_5 \mid \mathbf{d}) = 0.25 \cdot 0,44 + 0.5 \cdot 0,26 + 0.75 \cdot 0,29 = 0,46$